THE MANGA GUIDE™ TO

STATISTICS

SHIN TAKAHASHI
TREND-PRO, CO., LTD.

Ohmsha

no starch press

TABLE OF CONTENTS

6

LET'S LOOK AT THE RELATIONSHIP BETWEEN TWO VARIABLES 111

7

LET'S EXPLORE THE HYPOTHESIS TESTS 143

APPENDIX

LET'S CALCULATE USING EXCEL 191

INDEX 213

PREFACE

This is an introductory book on statistics. The intended readers are:

- Those who need to conduct data analysis for research or business
- Those who do not necessarily need to conduct data analysis now but are interested in getting an idea of what the world of statistics is like
- Those who have already acquired general knowledge of statistics and want to learn more

Statistics is one of the areas of mathematics most closely related to everyday life and business. Familiarizing yourself with statistics may come in handy in situations like:

- Estimating how many servings of fried noodles you can sell at a food stand you are planning to set up during a school festival
- Estimating whether you will be able to pass a certification exam
- Comparing the probability that a sick person will get better between a case in which medicine X is used and a case in which it is not used

This book consists of seven chapters. Basically, each chapter is organized in the following sections:

- Cartoon
- Text explanation to supplement the cartoon
- Exercise and answer
- Summary

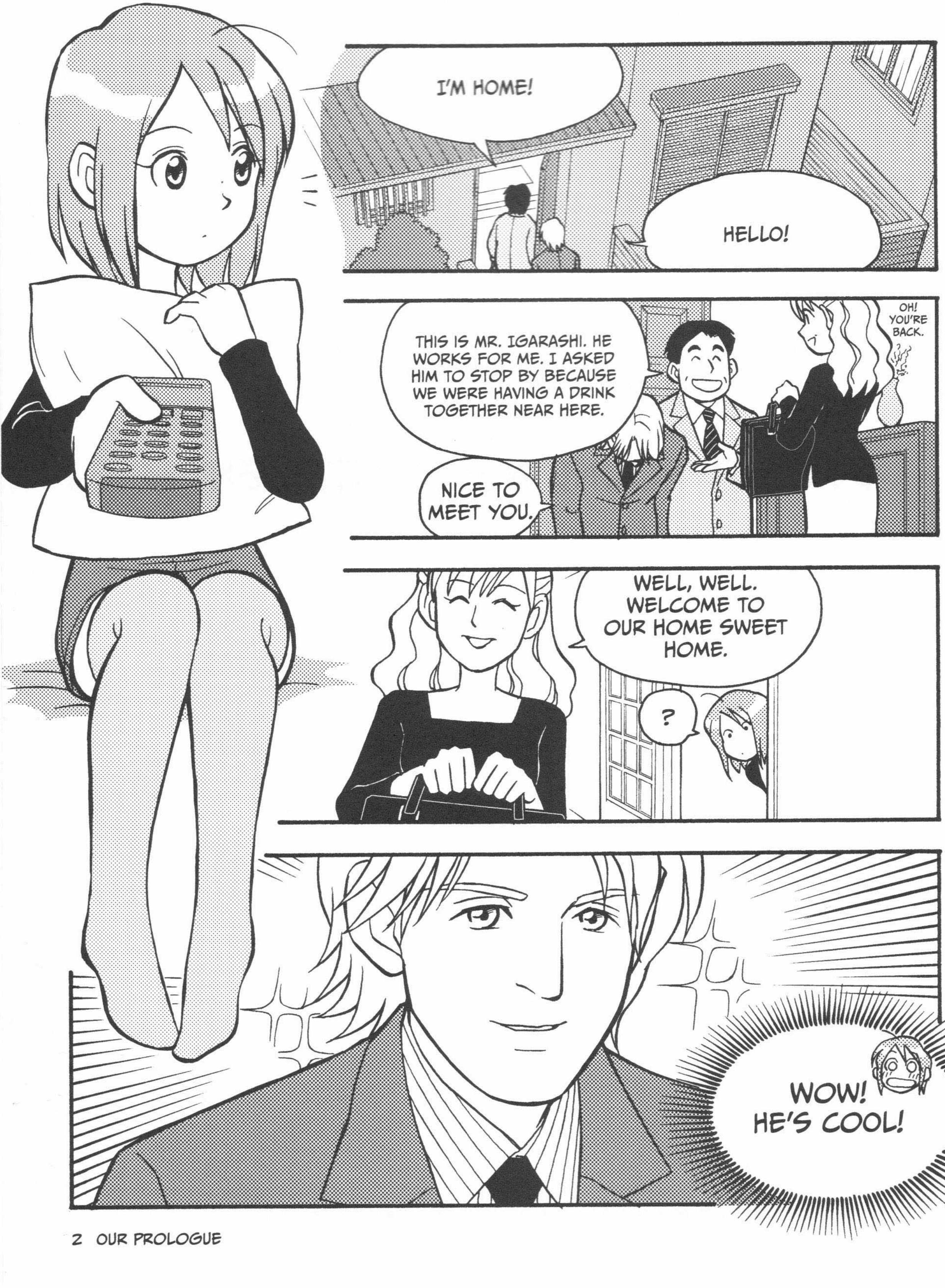
You can learn a lot by just reading the cartoon section, but deeper understanding and knowledge will be acquired if you read the other sections as well.

I would be very pleased if you start feeling that statistics is fun and useful after reading this book.

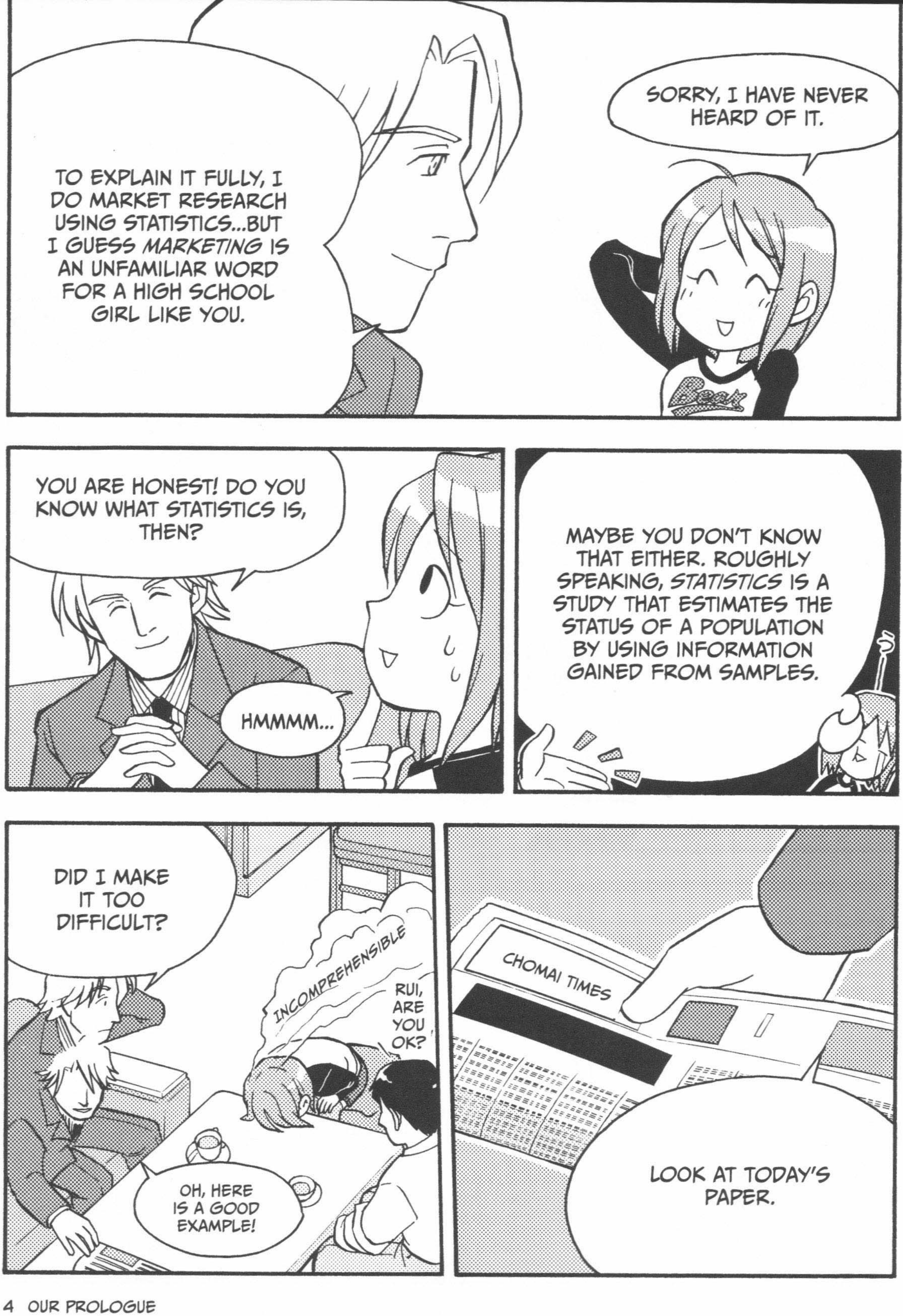
I would like to thank the staff in the development department of Ohmsha, Ltd., who offered me the opportunity to write this book. I would also like to thank TREND-PRO, Co., Ltd. for making my manuscript into a cartoon, the scenario writer, re_akino, and the illustrator, Iroha Inoue. Last but not least, I would like to thank Dr. Sakaori Fumitake of the College of Social Relations at Rikkyo University. He provided me with invaluable advice while I was preparing the manuscript for this book.

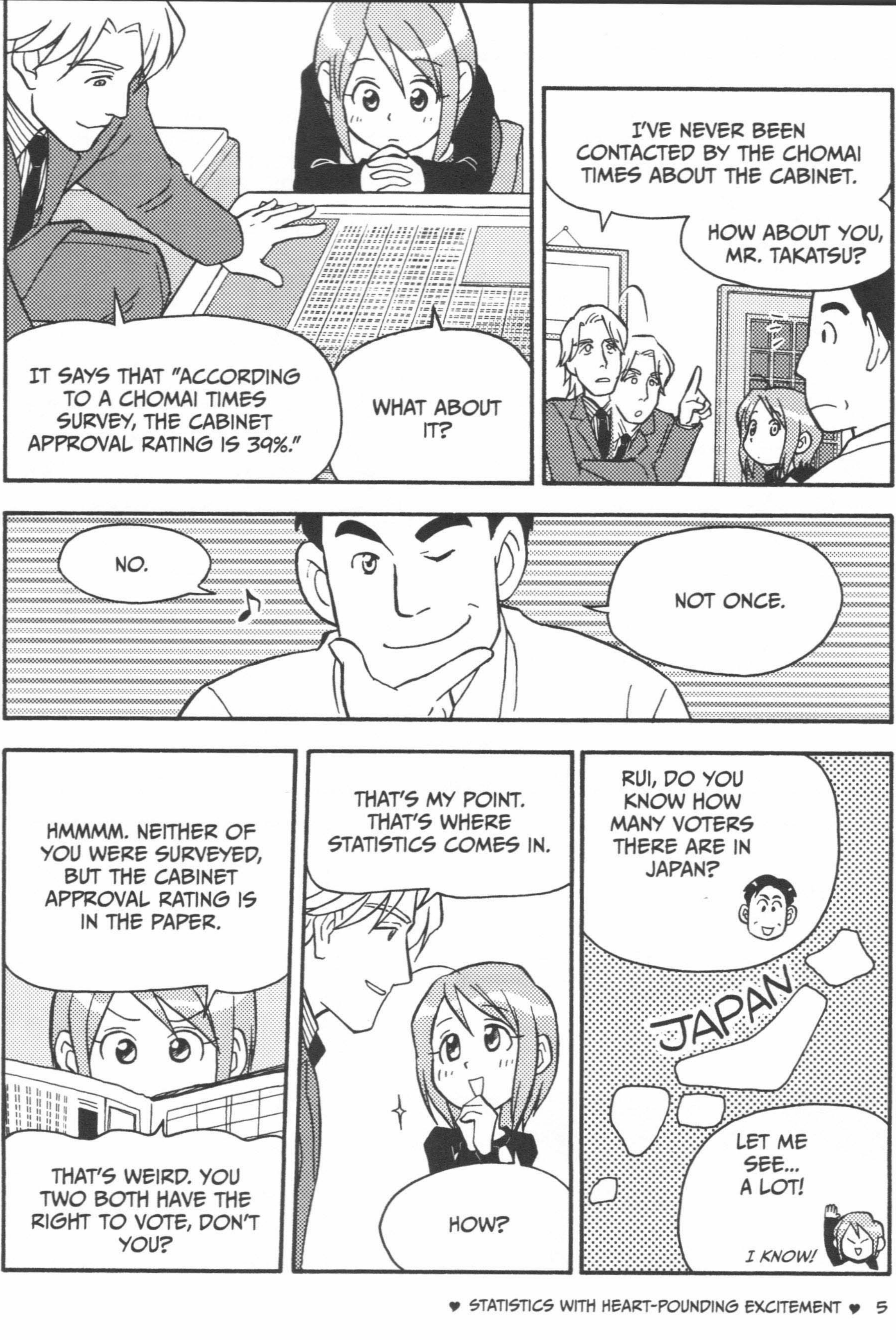
SHIN TAKAHASHI

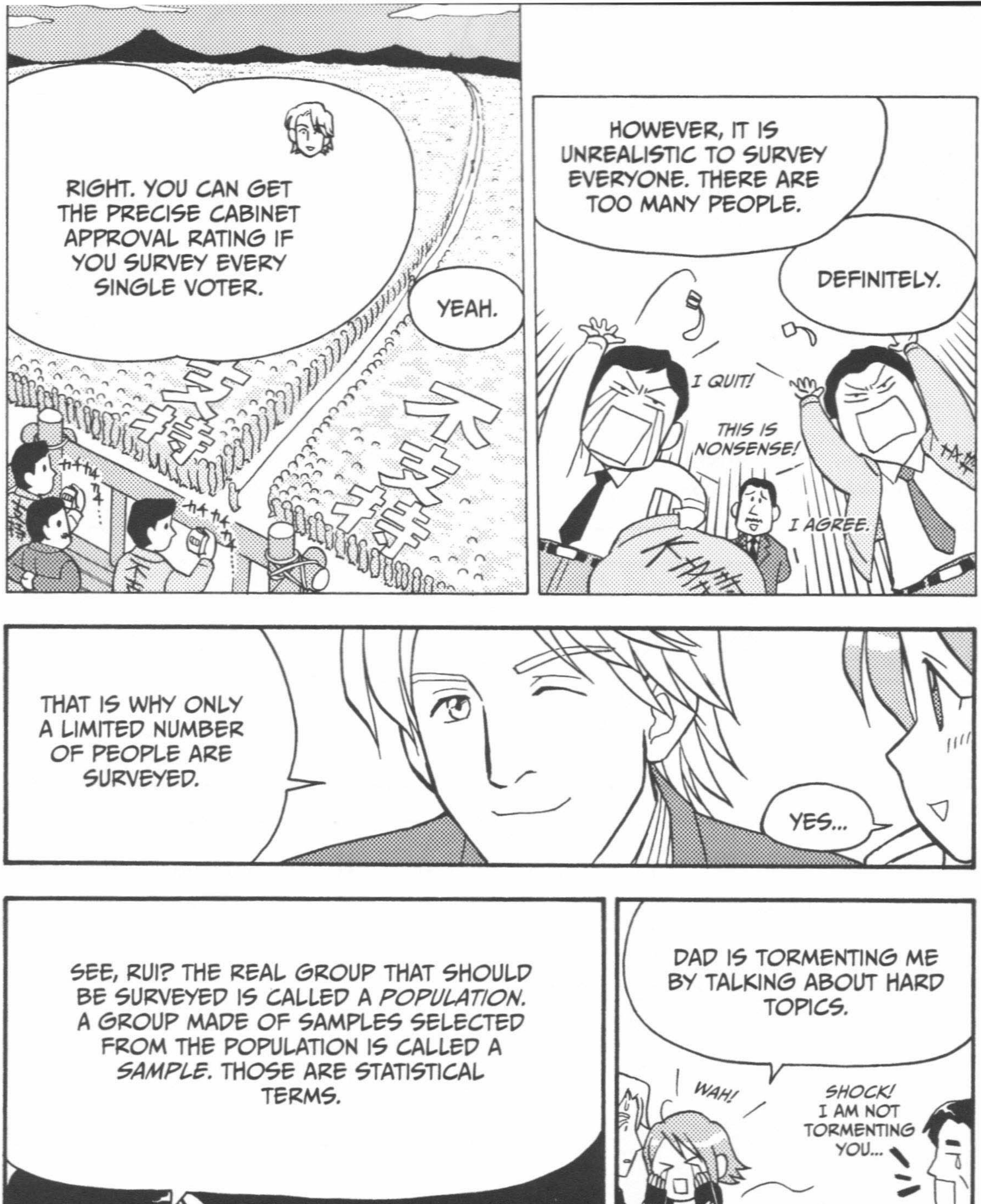
♥ OUR PROLOGUE: STATISTICS WITH HEART-POUNDING EXCITEMENT ♥

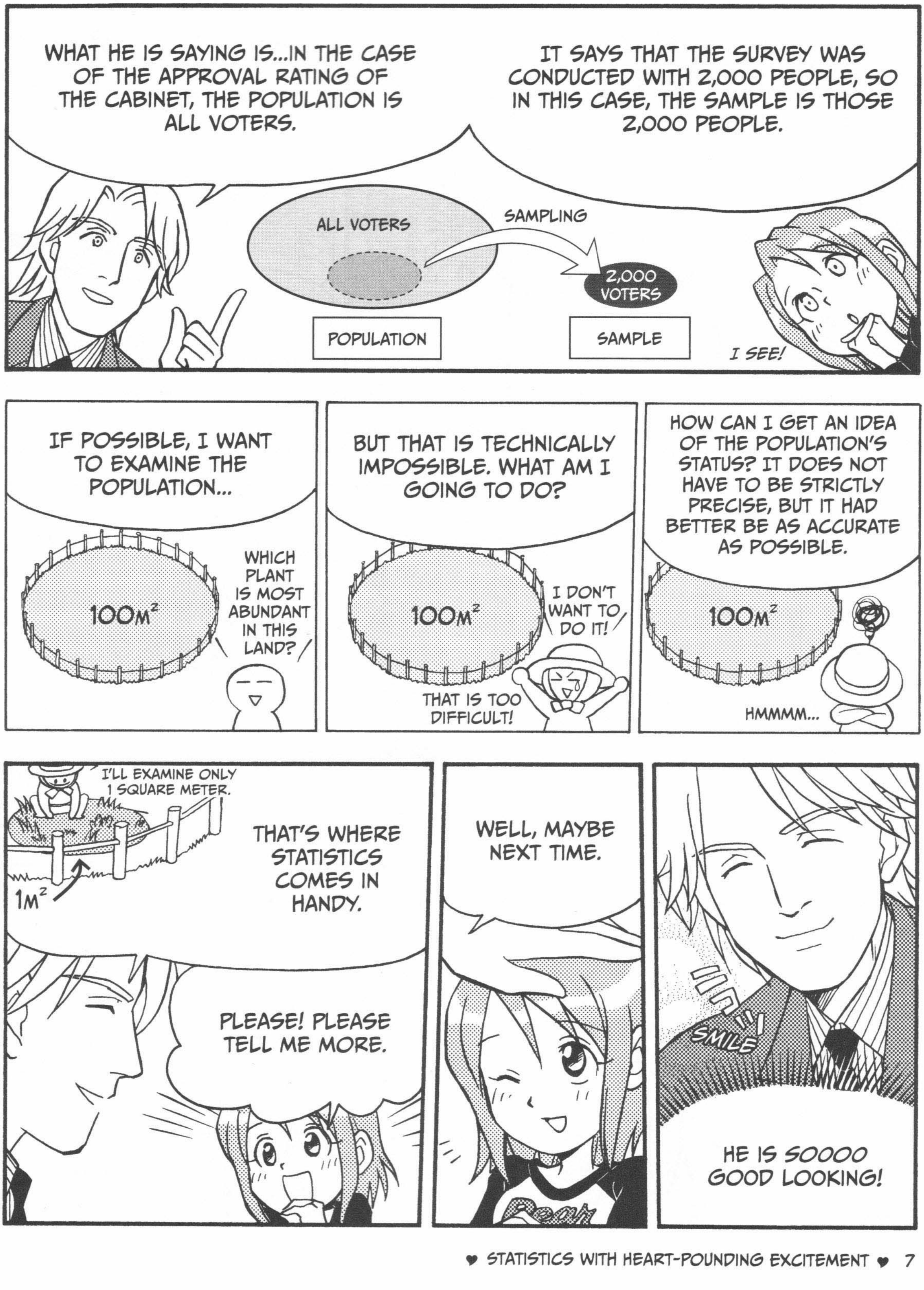

I'M HOME!
HELLO!
OH! YOU'RE BACK.
THIS IS MR. IGARASHI. HE WORKS FOR ME. I ASKED HIM TO STOP BY BECAUSE WE WERE HAVING A DRINK TOGETHER NEAR HERE.
NICE TO MEET YOU.
WELL, WELL. WELCOME TO OUR HOME SWEET HOME.
?
WOW! HE'S COOL!

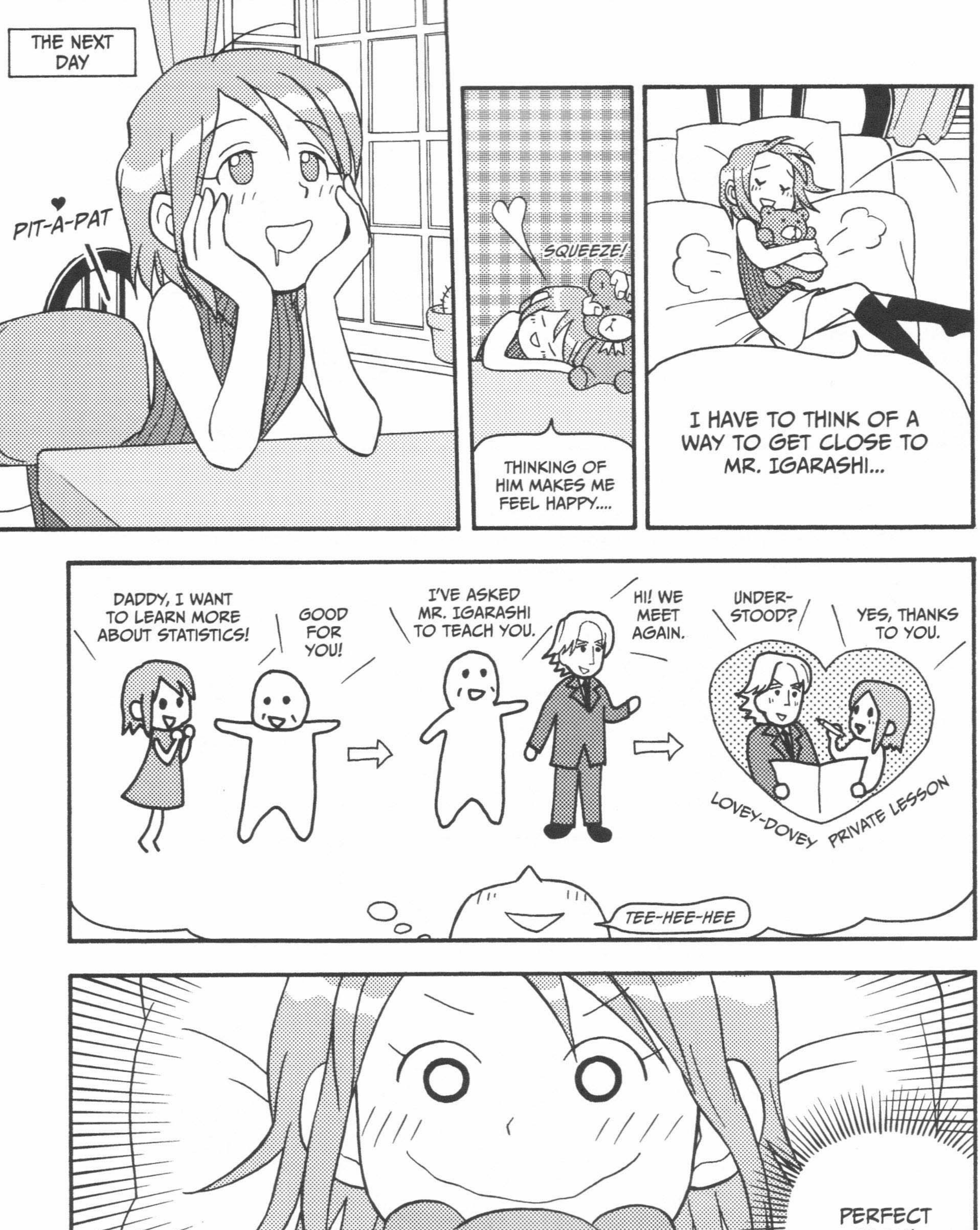

I'M HOME, RUI! SAY HELLO TO MR. IGARASHI. HE WORKS FOR ME.
GOOD EVENING.
PLEASE USE THE CUSHION.
PLEASE TAKE A SEAT.
TH... THANK YOU.
WOULD YOU LIKE A CUP OF COFFEE?
YOU HAVE A PRETTY DAUGHTER.
OH!
YOU FLATTER ME!
BUT I DO NOT DENY IT!
MR. IGARASHI, WHAT KIND OF WORK DO YOU DO?
WELL, AS I WORK FOR THE SAME COMPANY AS YOUR FATHER...
IN SHORT, I DO MARKETING.
MARKETING?
マーケティング??

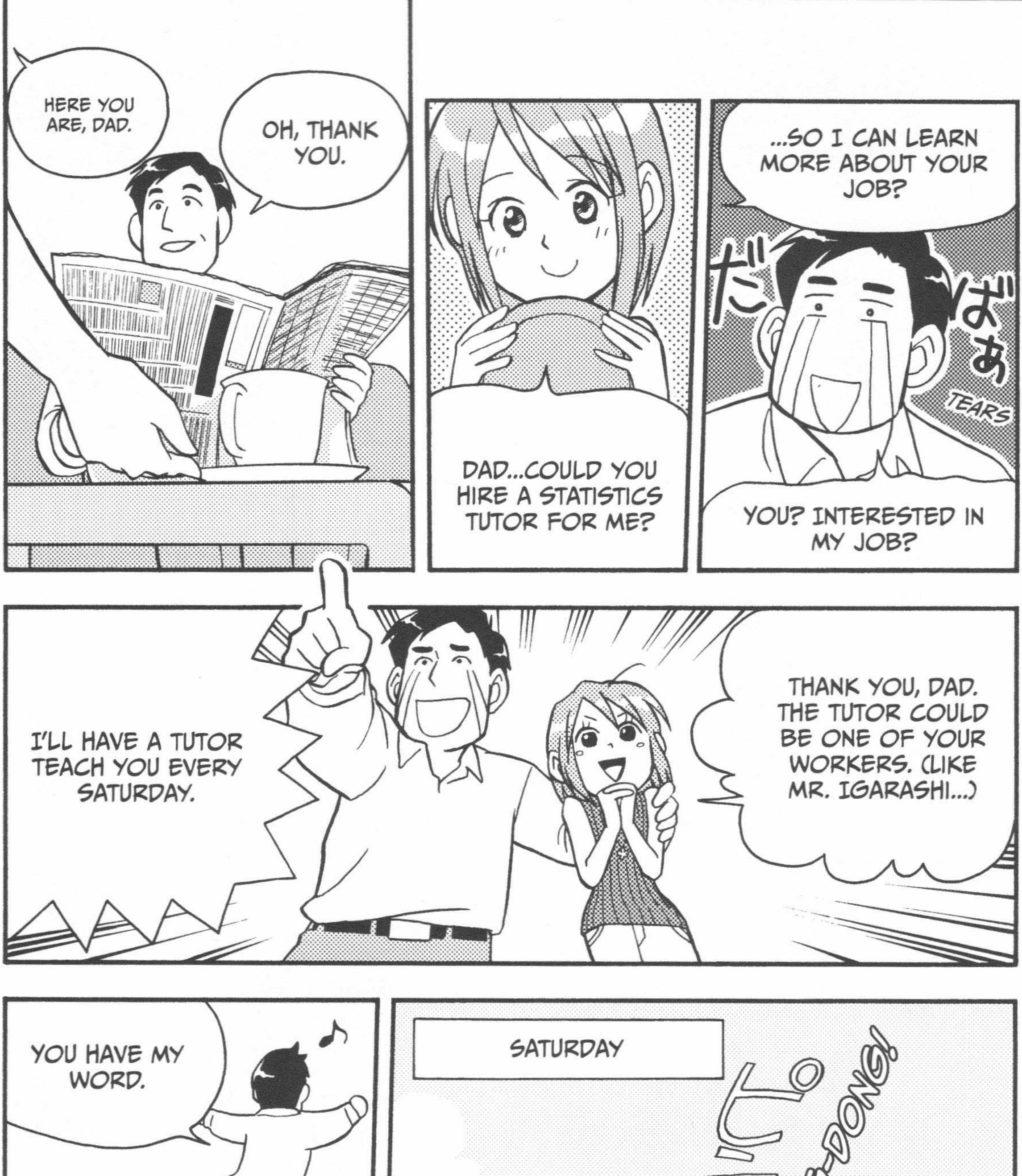

TO EXPLAIN IT FULLY, I DO MARKET RESEARCH USING STATISTICS...BUT I GUESS *MARKETING* IS AN UNFAMILIAR WORD FOR A HIGH SCHOOL GIRL LIKE YOU.
SORRY, I HAVE NEVER HEARD OF IT.
YOU ARE HONEST! DO YOU KNOW WHAT STATISTICS IS, THEN?
HMMMM...
MAYBE YOU DON'T KNOW THAT EITHER. ROUGHLY SPEAKING, *STATISTICS* IS A STUDY THAT ESTIMATES THE STATUS OF A POPULATION BY USING INFORMATION GAINED FROM SAMPLES.
DID I MAKE IT TOO DIFFICULT?
INCOMPREHENSIBLE
RUI, ARE YOU OK?
OH, HERE IS A GOOD EXAMPLE!
CHOMAI TIMES
LOOK AT TODAY'S PAPER.


IT SAYS THAT "ACCORDING TO A CHOMAI TIMES SURVEY, THE CABINET APPROVAL RATING IS 39%."
WHAT ABOUT IT?
I'VE NEVER BEEN CONTACTED BY THE CHOMAI TIMES ABOUT THE CABINET.
HOW ABOUT YOU, MR. TAKATSU?
NO.
NOT ONCE.
HMMMM. NEITHER OF YOU WERE SURVEYED, BUT THE CABINET APPROVAL RATING IS IN THE PAPER.
THAT'S WEIRD. YOU TWO BOTH HAVE THE RIGHT TO VOTE, DON'T YOU?
THAT'S MY POINT. THAT'S WHERE STATISTICS COMES IN.
HOW?
RUI, DO YOU KNOW HOW MANY VOTERS THERE ARE IN JAPAN?
JAPAN
LET ME SEE... A LOT!
I KNOW!


RIGHT. YOU CAN GET THE PRECISE CABINET APPROVAL RATING IF YOU SURVEY EVERY SINGLE VOTER.
YEAH.
支持
不支持
HOWEVER, IT IS UNREALISTIC TO SURVEY EVERYONE. THERE ARE TOO MANY PEOPLE.
DEFINITELY.
I QUIT!
THIS IS NONSENSE!
I AGREE.
THAT IS WHY ONLY A LIMITED NUMBER OF PEOPLE ARE SURVEYED.
YES...
SEE, RUI? THE REAL GROUP THAT SHOULD BE SURVEYED IS CALLED A *POPULATION*. A GROUP MADE OF SAMPLES SELECTED FROM THE POPULATION IS CALLED A *SAMPLE*. THOSE ARE STATISTICAL TERMS.
POPULATION?
SAMPLE?
DAD IS TORMENTING ME BY TALKING ABOUT HARD TOPICS.
WAH!
SHOCK! I AM NOT TORMENTING YOU...


WHAT HE IS SAYING IS...IN THE CASE OF THE APPROVAL RATING OF THE CABINET, THE POPULATION IS ALL VOTERS.
IT SAYS THAT THE SURVEY WAS CONDUCTED WITH 2,000 PEOPLE, SO IN THIS CASE, THE SAMPLE IS THOSE 2,000 PEOPLE.
ALL VOTERS
SAMPLING
2,000 VOTERS
POPULATION
SAMPLE
I SEE!
IF POSSIBLE, I WANT TO EXAMINE THE POPULATION...
100M²
WHICH PLANT IS MOST ABUNDANT IN THIS LAND?
BUT THAT IS TECHNICALLY IMPOSSIBLE. WHAT AM I GOING TO DO?
100M²
I DON'T WANT TO DO IT!
THAT IS TOO DIFFICULT!
HOW CAN I GET AN IDEA OF THE POPULATION'S STATUS? IT DOES NOT HAVE TO BE STRICTLY PRECISE, BUT IT HAD BETTER BE AS ACCURATE AS POSSIBLE.
100M²
HMMMM...
I'LL EXAMINE ONLY 1 SQUARE METER.
1M²
THAT'S WHERE STATISTICS COMES IN HANDY.
PLEASE! PLEASE TELL ME MORE.
WELL, MAYBE NEXT TIME.
SMILE
HE IS SOOOO GOOD LOOKING!


THE NEXT DAY
PIT-A-PAT
SQUEEZE!
THINKING OF HIM MAKES ME FEEL HAPPY....
I HAVE TO THINK OF A WAY TO GET CLOSE TO MR. IGARASHI...
DADDY, I WANT TO LEARN MORE ABOUT STATISTICS!
GOOD FOR YOU!
I'VE ASKED MR. IGARASHI TO TEACH YOU.
HI! WE MEET AGAIN.
UNDER-STOOD?
YES, THANKS TO YOU.
LOVEY-DOVEY PRIVATE LESSON
TEE-HEE-HEE
PERFECT PLAN!


HERE YOU ARE, DAD.
OH, THANK YOU.
DAD...COULD YOU HIRE A STATISTICS TUTOR FOR ME?
...SO I CAN LEARN MORE ABOUT YOUR JOB?
だばぁ
TEARS
YOU? INTERESTED IN MY JOB?
I'LL HAVE A TUTOR TEACH YOU EVERY SATURDAY.
THANK YOU, DAD. THE TUTOR COULD BE ONE OF YOUR WORKERS. (LIKE MR. IGARASHI...)
YOU HAVE MY WORD.
YES!
SATURDAY
ピンポーン
DING-DONG!
THANKS FOR COMING. COME IN.


HE'S HERE!
RUI! YOUR TEACHER IS HERE.
COMING!
タッタッタ…
STOMP STOMP STOMP
HI...

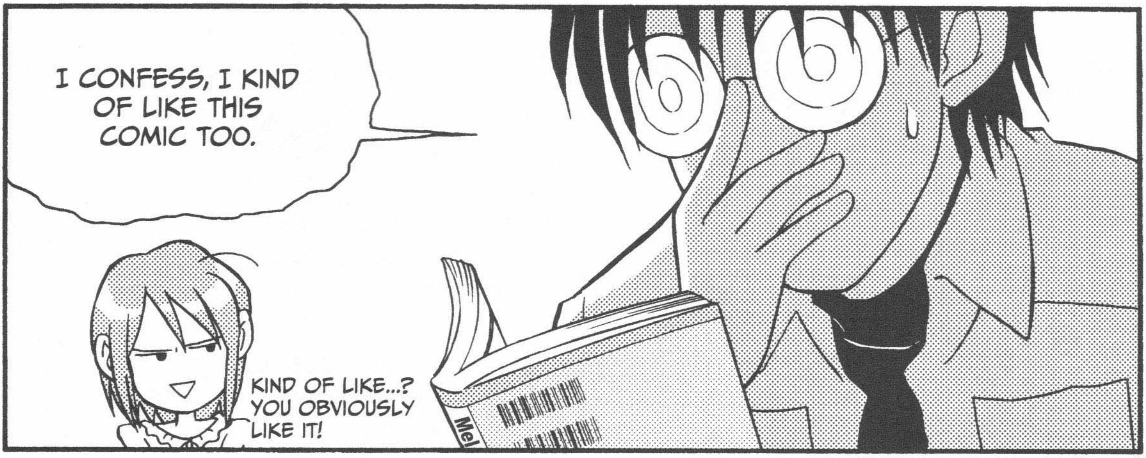

ボロッ...
RUI, THIS IS MY EMPLOYEE, MAMORU YAMAMOTO.
HOW DO YOU DO.
WHO IS THIS GUY?!
ガーン
DAD...ISN'T MR. IGARASHI COMING?
IGARASHI? MAMORU LIVES CLOSER TO OUR HOUSE. HE IS GOOD AT TEACHING, TOO.
STUDY HARD!
HO HO HO!

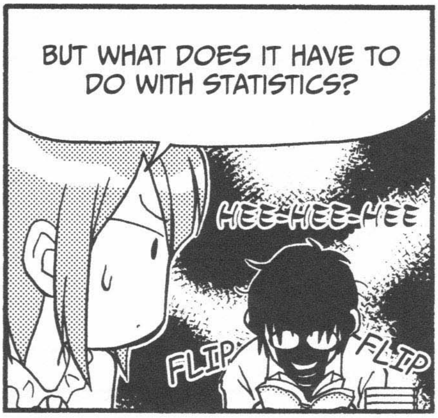

SHALL WE BEGIN, RUI?
THIS IS A *NIGHTMARE.*
UNNNGH...
MR. IGARASHI, I WORKED HARD TO MASTER STATISTICS!
GREAT! WHY DON'T YOU WORK WITH ME.
UM... RUI?
THIS IS NOT WHAT I HAD IN MIND...
I HAVE AN IDEA!
?
I'LL USE HIM TO STUDY STATISTICS, AND THEN APPROACH MR. IGARASHI!
NEVER GIVE UP!
LET'S START RIGHT NOW!
OH... OK.
THUS THE LESSON BEGAN.

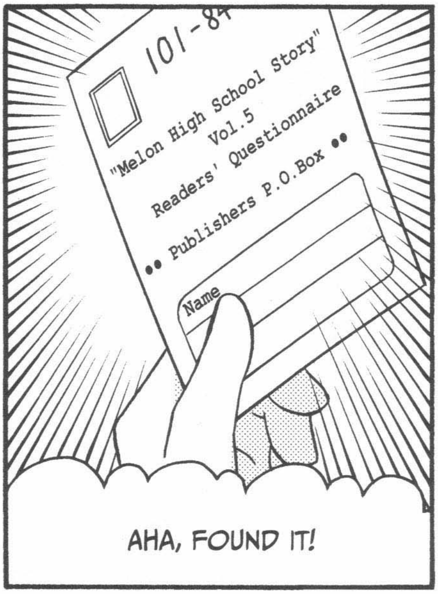
1

DETERMINING DATA TYPES

1. CATEGORICAL DATA AND NUMERICAL DATA







Melon High School Story Vol. 5 Reader Questionnaire

Q1. What is your impression of *Melon High School Story* Vol. 5?

1. Very fun
2. Rather fun
3. Average
4. Rather boring
5. Very boring

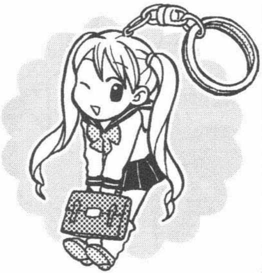
Q2. Sex

1. Female
2. Male

Q3. Age ________ years old

Q4. How many comics do you purchase per month? ________ titles

A Rina keychain will be given away to 30 lucky winners among those who send back this questionnaire!

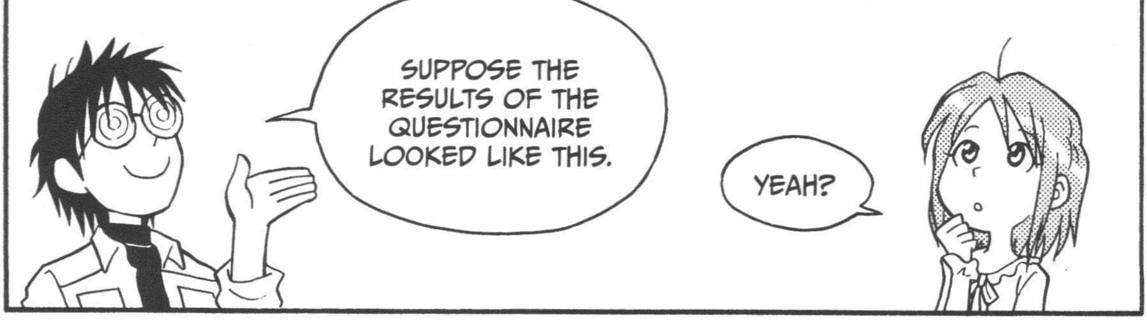


THANK YOU FOR YOUR CONTRIBUTION. YOUR VALUABLE OPINION WILL BE REFLECTED IN OUR FUTURE PUBLICATIONS.

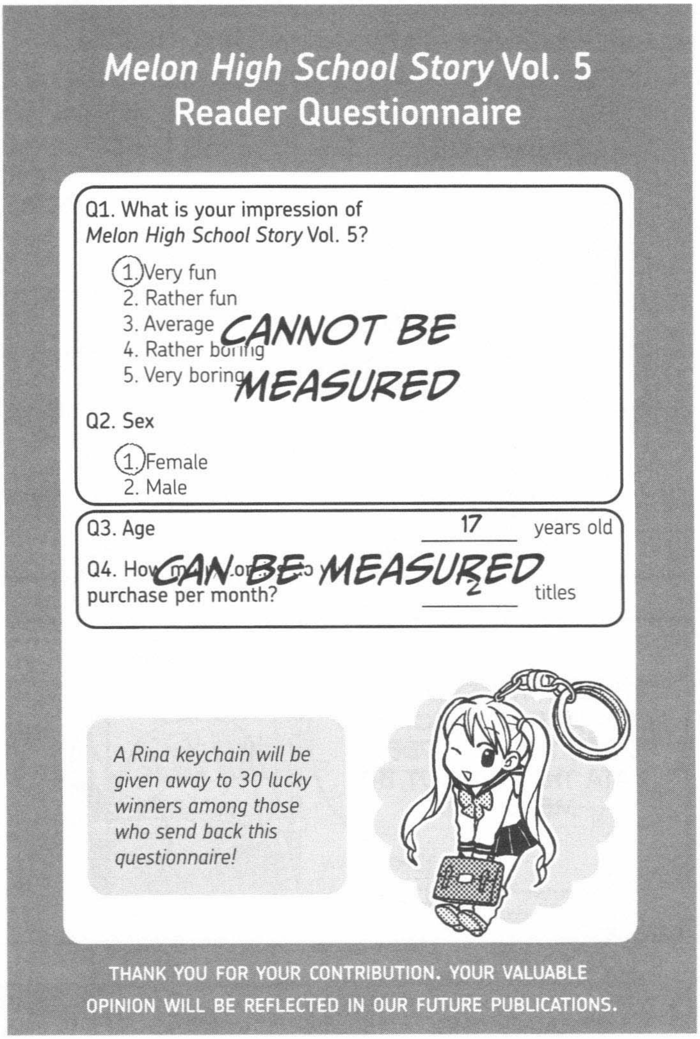

WHAT ABOUT THIS QUESTIONNAIRE?
IF YOU ANSWER THAT QUESTIONNAIRE...
FLASH!
101-8460
I BELIEVED YOU!
I'LL DO MY BEST!
I MADE THIS.
BUT I LIKE YOU!
YOU MIGHT BE ABLE TO GET A RINA KEYCHAIN! (IF YOU ARE LUCKY ENOUGH...)
GOD, I WISH I COULD WIN ONE...
YOU DORK!
NOW, BACK TO STUDYING...
SO SUDDENLY?!
JUST ANSWER THAT QUESTIONNAIRE.
ALL RIGHT.
DONE!
SCRIBBLE SCRIBBLE
GOOD.

QUESTIONNAIRE RESULTS

RESPONDENT	Q1 YOUR IMPRESSION OF *MELON HIGH SCHOOL STORY*	Q2 SEX	Q3 AGE	Q4 COMIC BOOK PURCHASES PER MONTH
RUI	VERY FUN	FEMALE	17	2
A	RATHER FUN	FEMALE	17	1
B	AVERAGE	MALE	18	5
C	RATHER BORING	MALE	22	7
D	RATHER FUN	FEMALE	25	4
E	VERY BORING	MALE	20	3
F	VERY FUN	FEMALE	16	1
G	RATHER FUN	FEMALE	17	2
H	AVERAGE	MALE	18	0
I	AVERAGE	FEMALE	21	3
...	...	...	...	...




PERSONALLY, I AM EXTREMELY ANXIOUS TO KNOW IF *MELON HIGH SCHOOL STORY* WAS WELL RECEIVED BY THE READERS!
YOU "MELON HIGH SCHOOL" FREAK!
THUMP
BUT LET'S STAY FOCUSED. THE POINT OF TODAY'S LESSON IS TO CATEGORIZE DATA.
WHAT DO YOU MEAN?
WHEN YOU LOOK AT DATA, THERE ARE TWO TYPES:
DATA THAT YOU CAN MEASURE AND DATA THAT YOU CANNOT MEASURE.
CAN BE MEASURED
CANNOT BE MEASURED



Melon High School Story Vol. 5
Reader Questionnaire

Q1. What is your impression of *Melon High School Story* Vol. 5?

[✓ 1. Very fun]
2. Rather fun
3. Average
4. Rather boring
5. Very boring

Q2. Sex

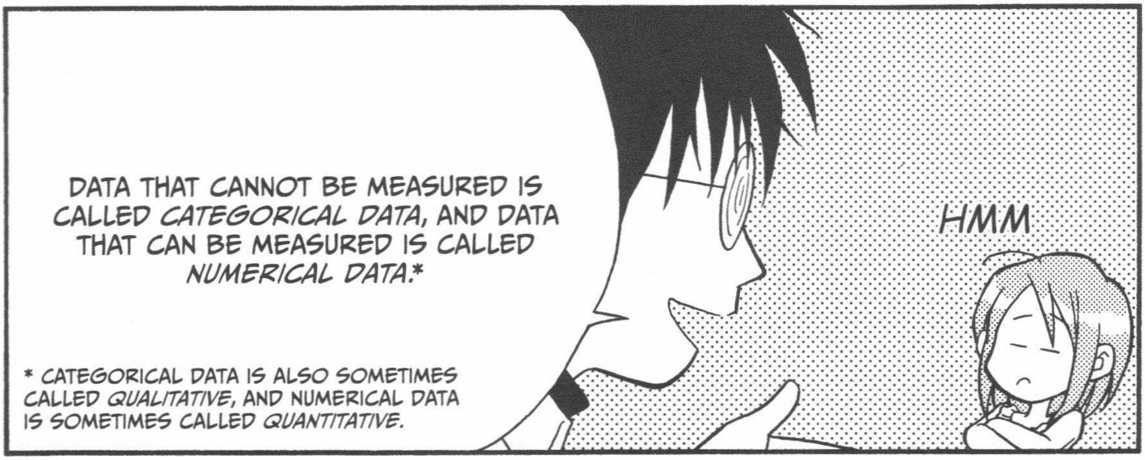
[✓ 1. Female]
2. Male

Q3. Age 17 years old

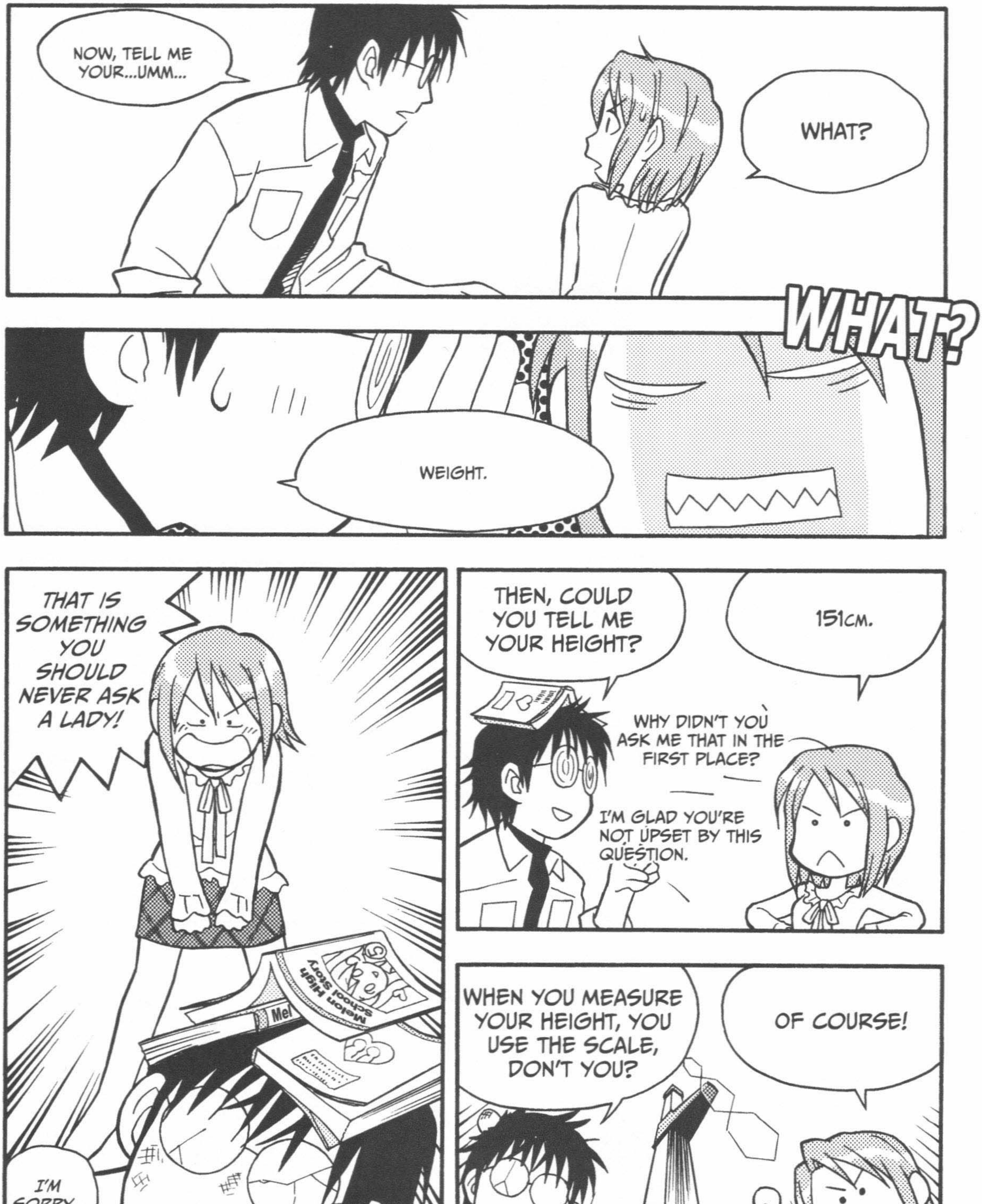
Q4. How many [illegible] do you purchase per month? 2 titles

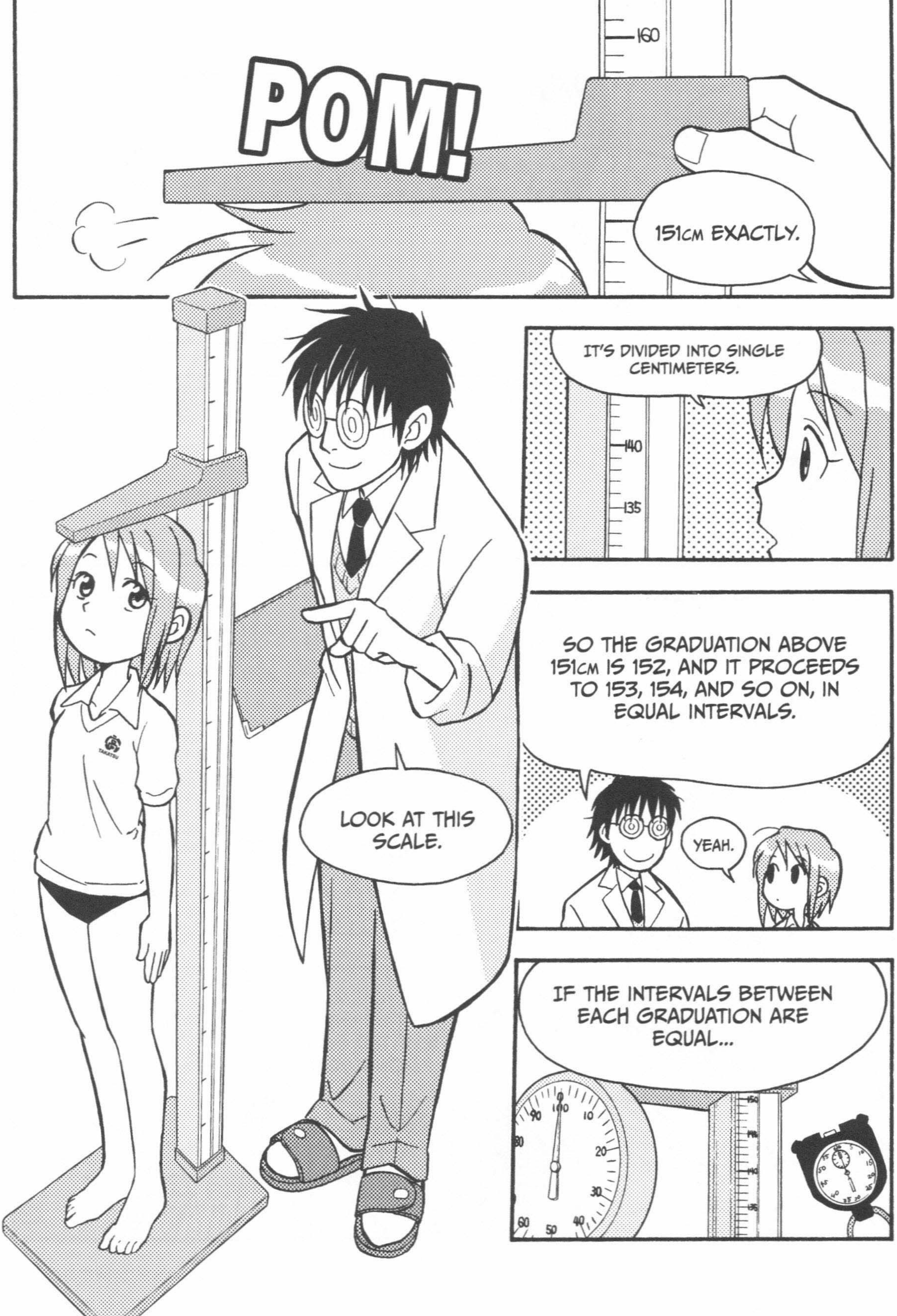
A Rina keychain will be given away to 30 lucky winners among those who send back this questionnaire!

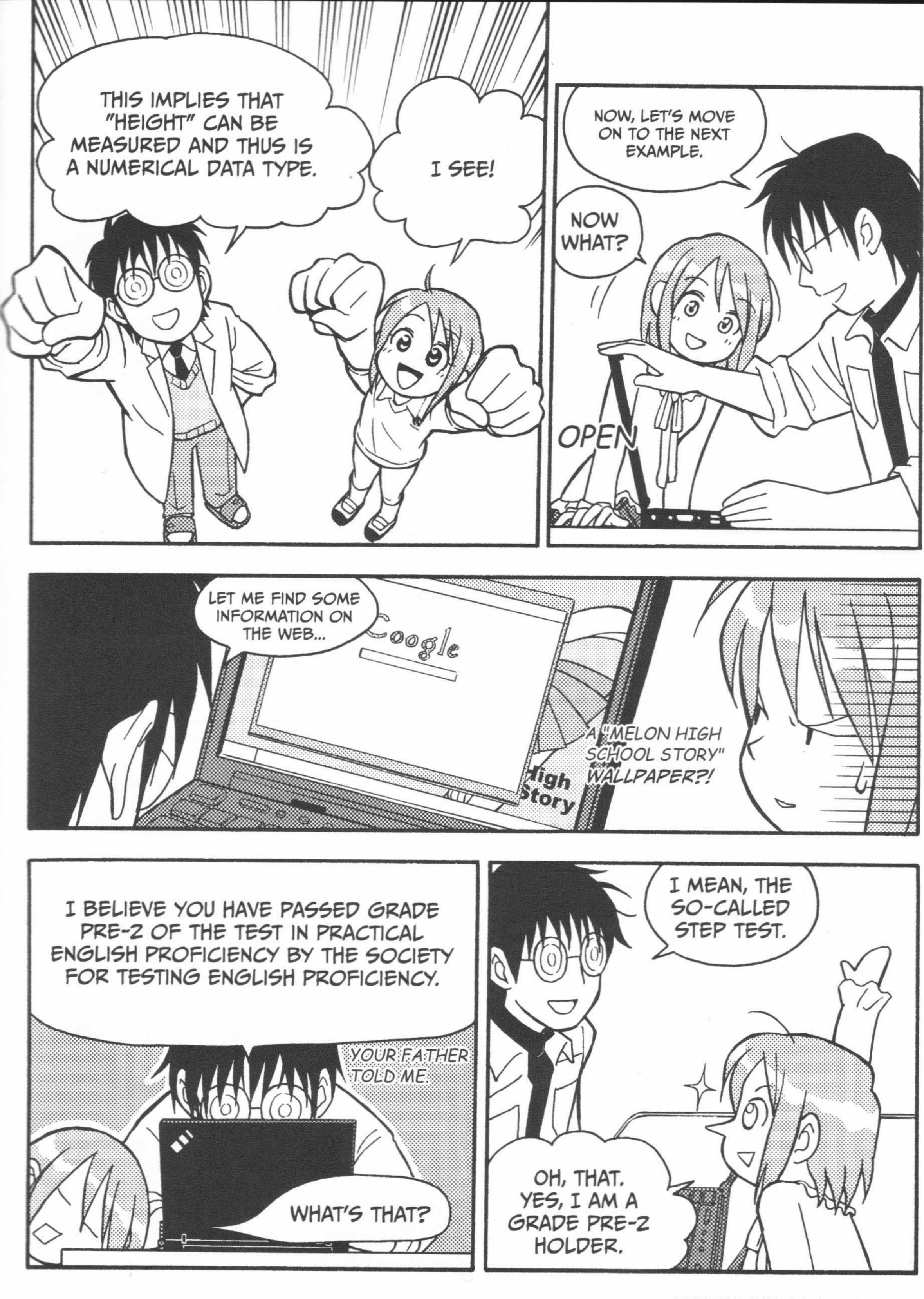
THANK YOU FOR YOUR CONTRIBUTION. YOUR VALUABLE OPINION WILL BE REFLECTED IN OUR FUTURE PUBLICATIONS.

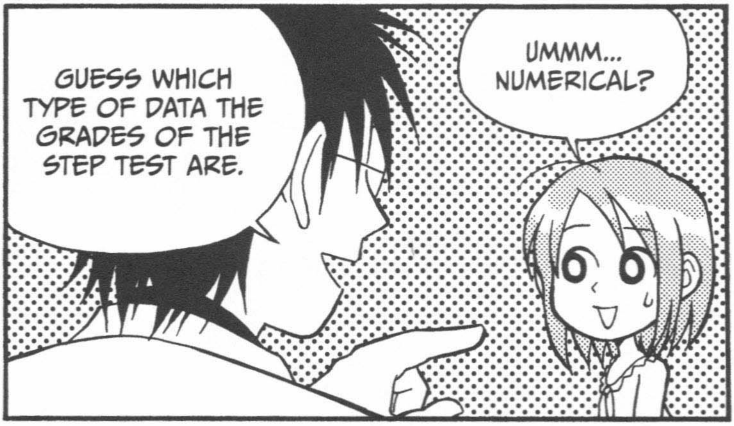


2. AN EXAMPLE OF TRICKY CATEGORICAL DATA


NOW, TELL ME YOUR...UMM...
WHAT?
WEIGHT.
WHAT?
THAT IS SOMETHING YOU SHOULD NEVER ASK A LADY!
Melon High School Story
Mel
I'M SORRY...
THEN, COULD YOU TELL ME YOUR HEIGHT?
151CM.
WHY DIDN'T YOU ASK ME THAT IN THE FIRST PLACE?
I'M GLAD YOU'RE NOT UPSET BY THIS QUESTION.
WHEN YOU MEASURE YOUR HEIGHT, YOU USE THE SCALE, DON'T YOU?
OF COURSE!


160
POM!
151CM EXACTLY.
LOOK AT THIS SCALE.
IT'S DIVIDED INTO SINGLE CENTIMETERS.
140
135
SO THE GRADUATION ABOVE 151CM IS 152, AND IT PROCEEDS TO 153, 154, AND SO ON, IN EQUAL INTERVALS.
YEAH.
IF THE INTERVALS BETWEEN EACH GRADUATION ARE EQUAL...

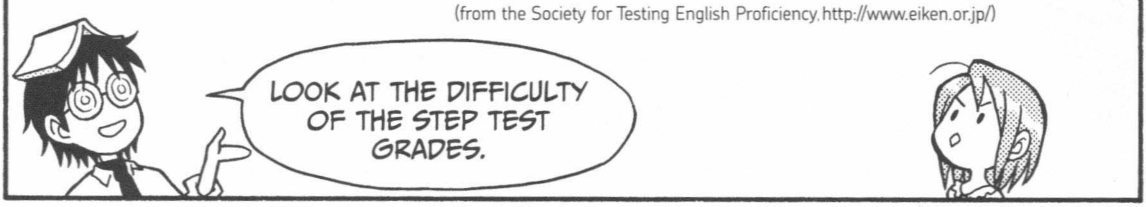

THIS IMPLIES THAT "HEIGHT" CAN BE MEASURED AND THUS IS A NUMERICAL DATA TYPE.
I SEE!
NOW, LET'S MOVE ON TO THE NEXT EXAMPLE.
NOW WHAT?
OPEN
LET ME FIND SOME INFORMATION ON THE WEB...
Google
High School Story
A "MELON HIGH SCHOOL STORY" WALLPAPER?!
I BELIEVE YOU HAVE PASSED GRADE PRE-2 OF THE TEST IN PRACTICAL ENGLISH PROFICIENCY BY THE SOCIETY FOR TESTING ENGLISH PROFICIENCY.
YOUR FATHER TOLD ME.
WHAT'S THAT?
I MEAN, THE SO-CALLED STEP TEST.
OH, THAT. YES, I AM A GRADE PRE-2 HOLDER.

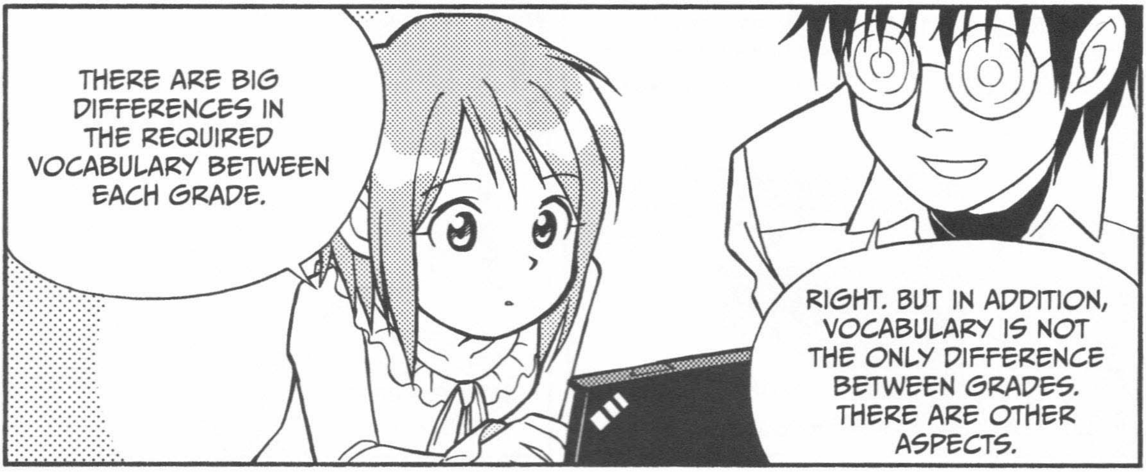


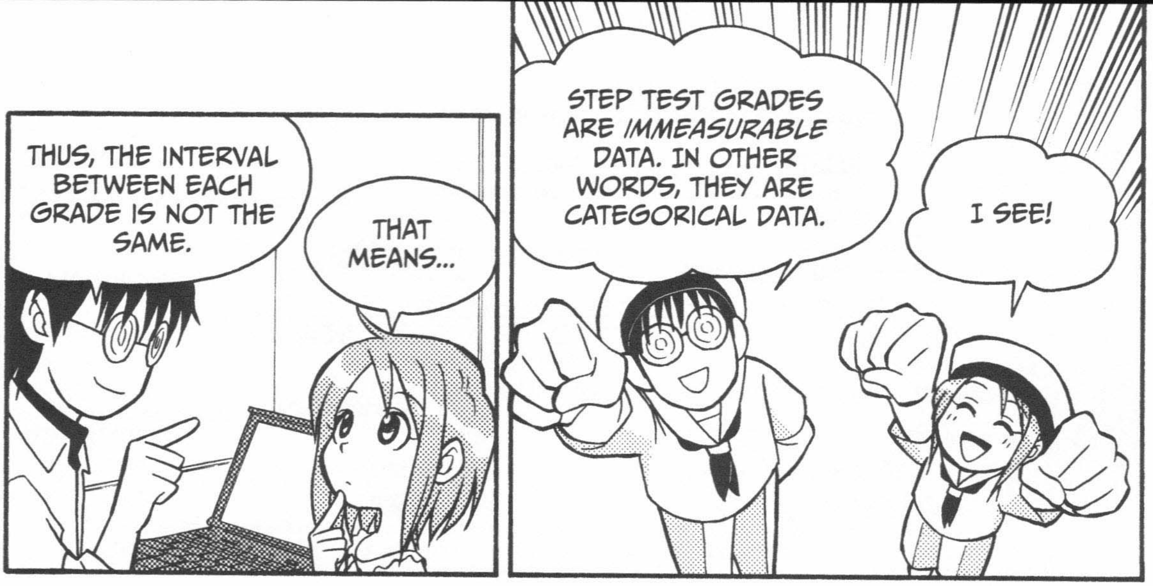
THE STEP TEST GRADES

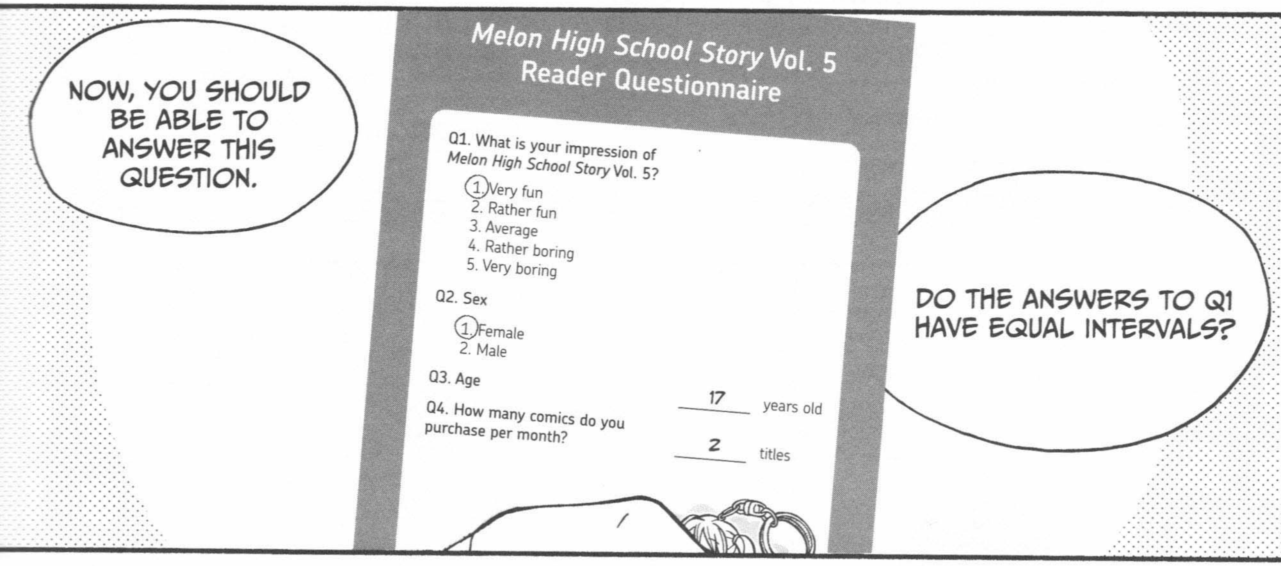
Grade	Requirements
Grade 1	Advanced university graduate level, vocabulary 10,000–15,000 words
Grade 2	High school graduate level, vocabulary 5,100 words
Grade 3	Junior high school graduate level, vocabulary 2,100 words
Grade 4	Intermediate junior high school level, vocabulary 1,300 words
Grade 5	Beginner junior high school level, vocabulary 600 words

(from the Society for Testing English Proficiency, http://www.eiken.or.jp/)









Melon High School Story Vol. 5
Reader Questionnaire

Q1. What is your impression of *Melon High School Story* Vol. 5?

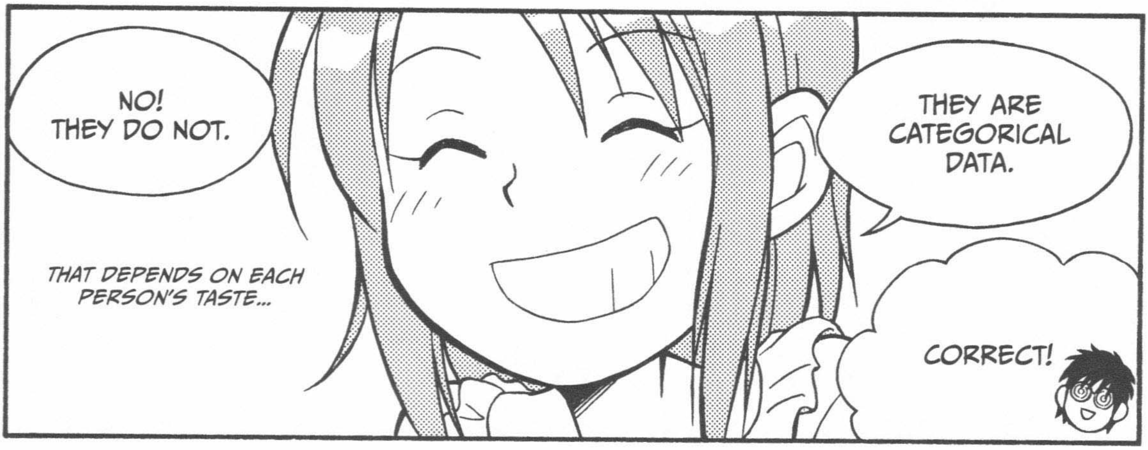
[✓ 1. Very fun]
2. Rather fun
3. Average
4. Rather boring
5. Very boring

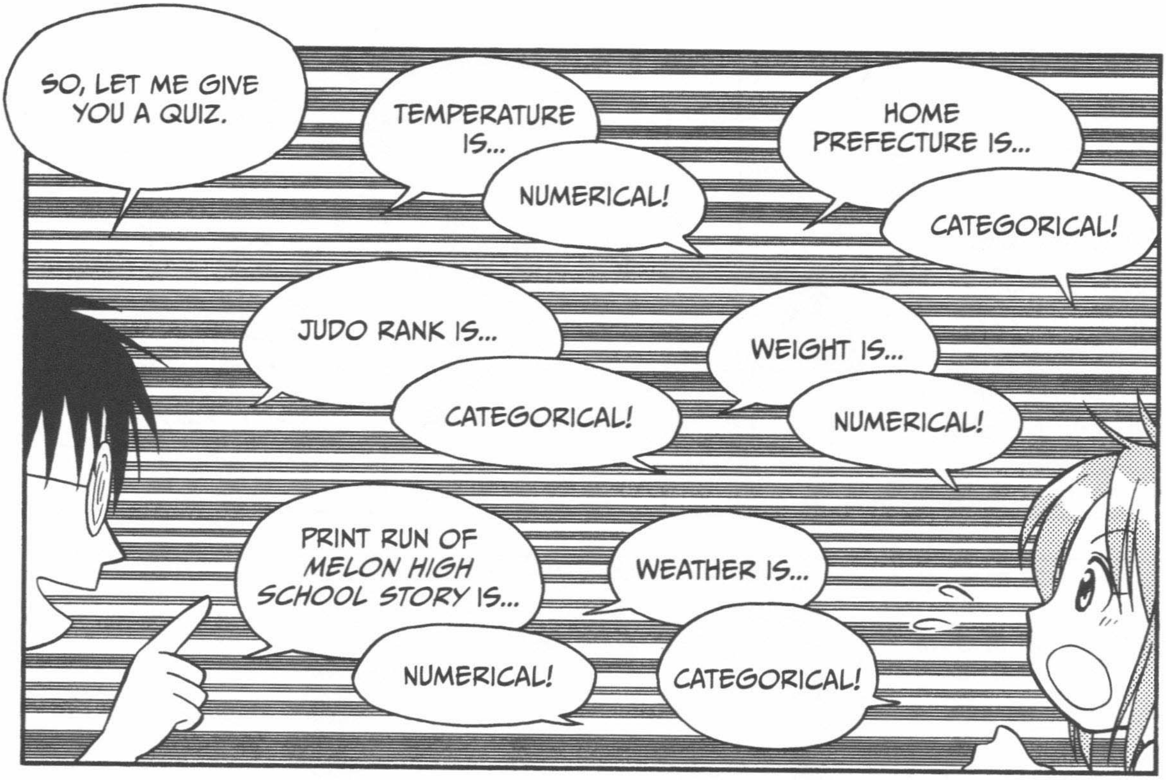
Q2. Sex

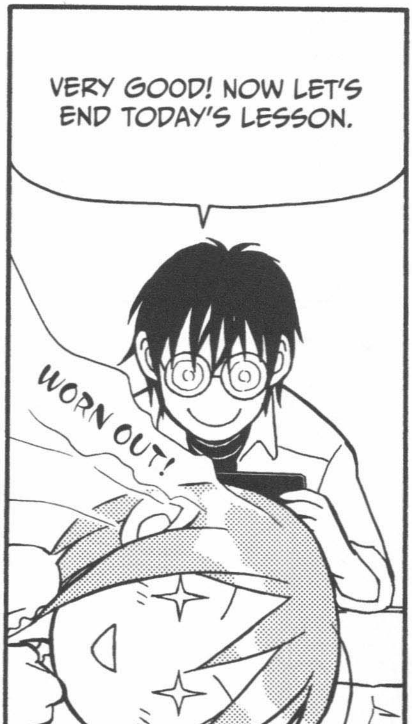
[✓ 1. Female]
2. Male

Q3. Age 17 years old

Q4. How many comics do you purchase per month? 2 titles




SO, LET ME GIVE YOU A QUIZ.
TEMPERATURE IS...
NUMERICAL!
HOME PREFECTURE IS...
CATEGORICAL!
JUDO RANK IS...
CATEGORICAL!
WEIGHT IS...
NUMERICAL!
PRINT RUN OF *MELON HIGH SCHOOL STORY* IS...
NUMERICAL!
WEATHER IS...
CATEGORICAL!


VERY GOOD! NOW LET'S END TODAY'S LESSON.
WORN OUT!


THANK YOU FOR THE LESSON.
SEE YOU NEXT WEEK.


BY THE WAY...

igh School Story Vol. 5
Reader Questionnaire

Q1. What is your impression of *Melon High School Story Vol. 5*?
[✓ 1. Very fun]
2. Rather fun
3. Average
4. Rather boring
5. Very boring

Q2. Sex
[✓ 1. Female]
2. Male

Q3. Age 17 years old

Q4. How many comics do you purchase per month? 2 titles

A Rina keychain will be given away to 30 lucky winners among those who send back this questionnaire!

THANK Y
OPINION W CONTRIBUTION. YOUR VALUABLE
D IN OUR FUTURE PUBLICATIONS.

3. HOW MULTIPLE-CHOICE ANSWERS ARE HANDLED IN PRACTICE

As mentioned on page 25, the multiple-choice answers for the first question of the readers' questionnaire are categorical data. However, in practice, it is possible to handle such data as numerical data when processing consumer questionnaires and so on. Some examples are below.

Very fun	⇨	5 points
Rather fun	⇨	4 points
Average	⇨	3 points
Rather boring	⇨	2 points
Very boring	⇨	1 point

Very fun	⇨	2 points
Rather fun	⇨	1 point
Average	⇨	0 points
Rather boring	⇨	–1 points
Very boring	⇨	–2 points

The same data is handled differently in theory and in practice. Keep in mind that data may be categorized differently in different situations.

EXERCISE AND ANSWER

EXERCISE

Determine whether the data in the following table is categorical data or numerical data.

Respondent	Blood type	Opinion on sports drink *X*	Comfortable air conditioning temperature (°C)	100m track race record (seconds)
Mr./Ms. A	B	Not good	25	14.1
Mr./Ms. B	A	Good	24	12.2
Mr./Ms. C	AB	Good	25	17.0
Mr./Ms. D	O	Average	27	15.6
Mr./Ms. E	A	Not good	24	18.4
...	...	...	...	...

ANSWER

Blood type and opinion on sports drink *X* are examples of categorical data. Comfortable air conditioning temperature and 100m track race record are examples of numerical data.

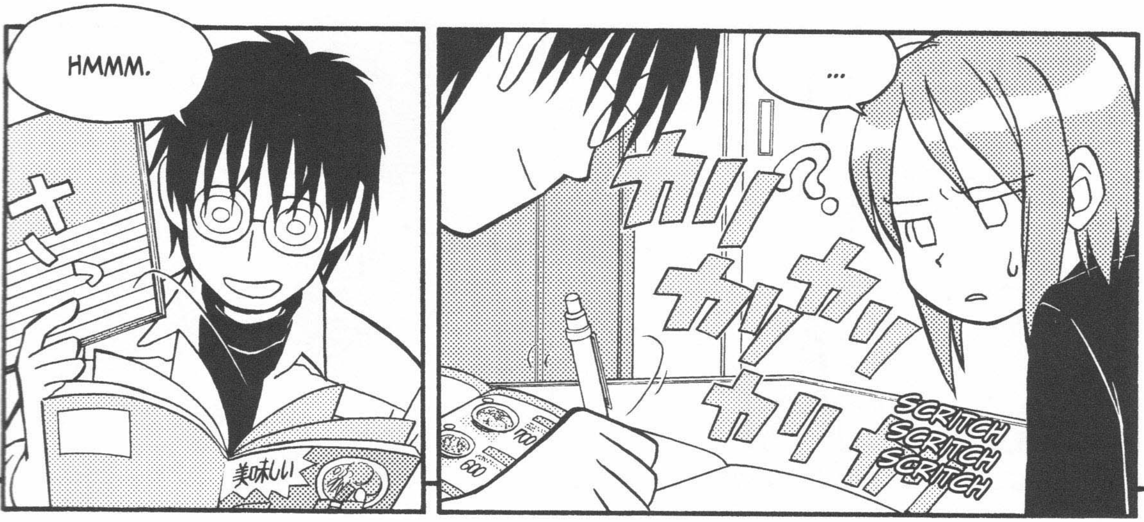
SUMMARY

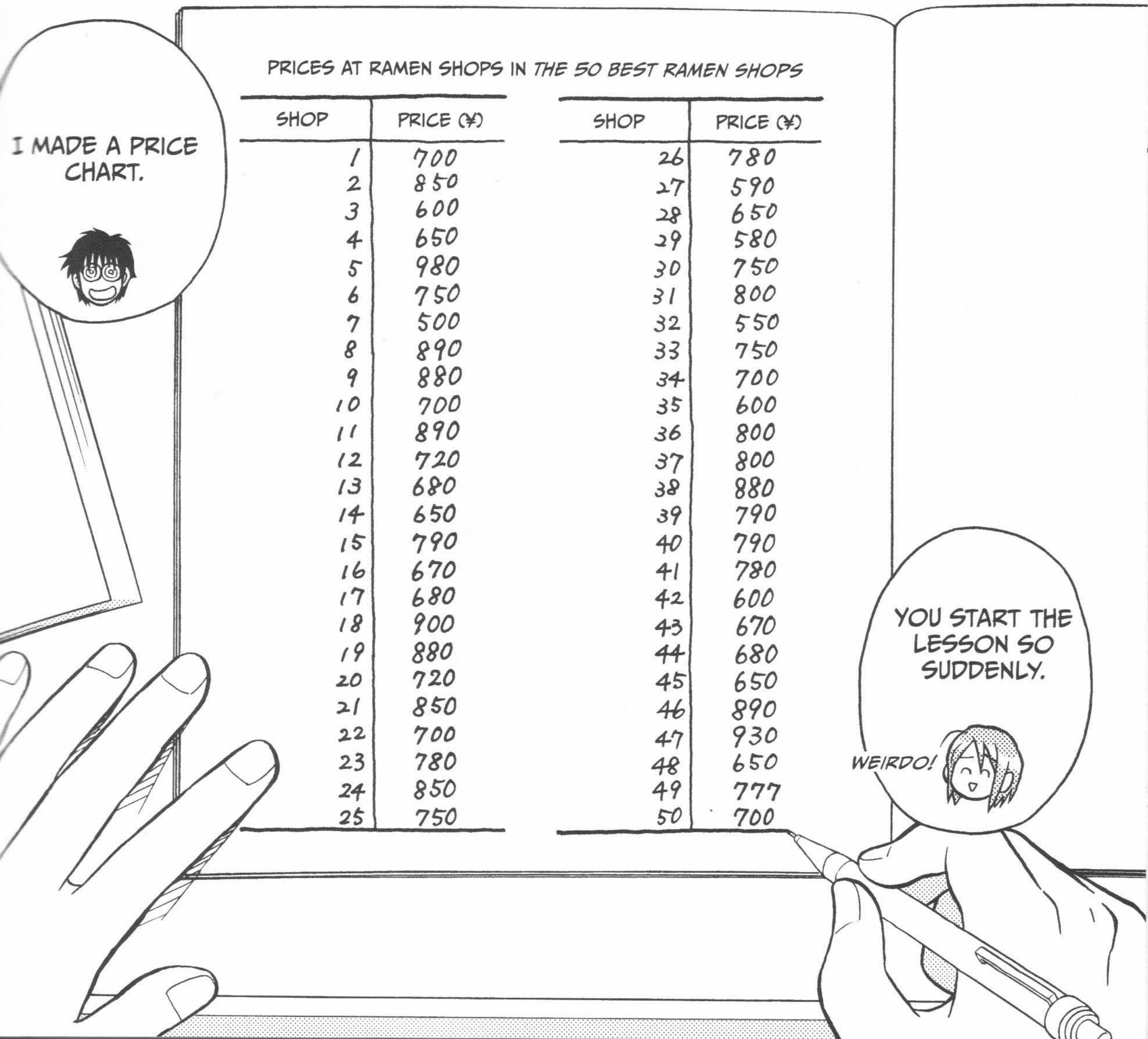
- Data is classified as categorical data or numerical data.
- Some data, such as "very fun" or "very boring," is theoretically categorical data. However, in practice, it is possible to treat it as numerical data.

2

GETTING THE BIG PICTURE: UNDERSTANDING NUMERICAL DATA

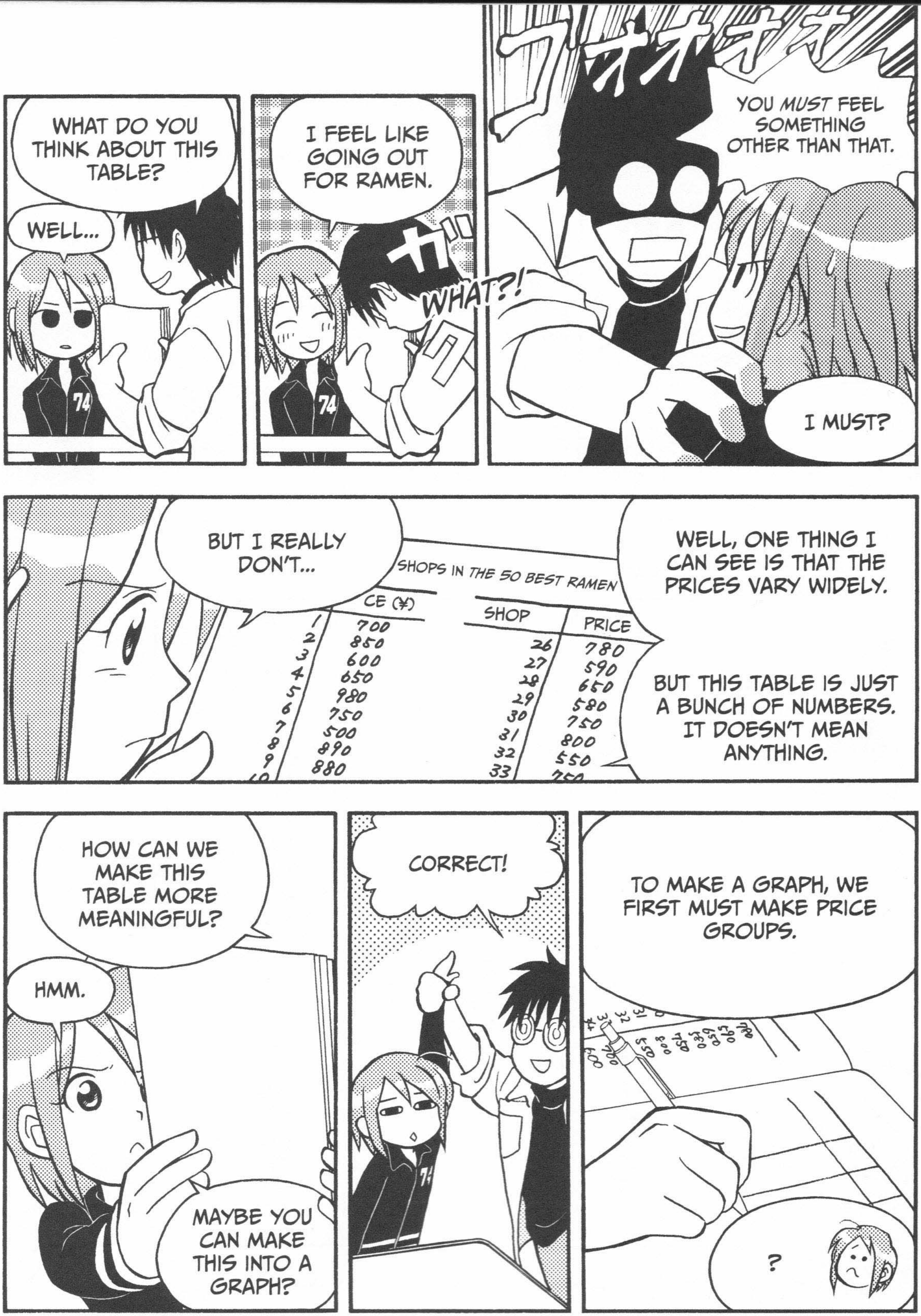
1. FREQUENCY DISTRIBUTION TABLES AND HISTOGRAMS

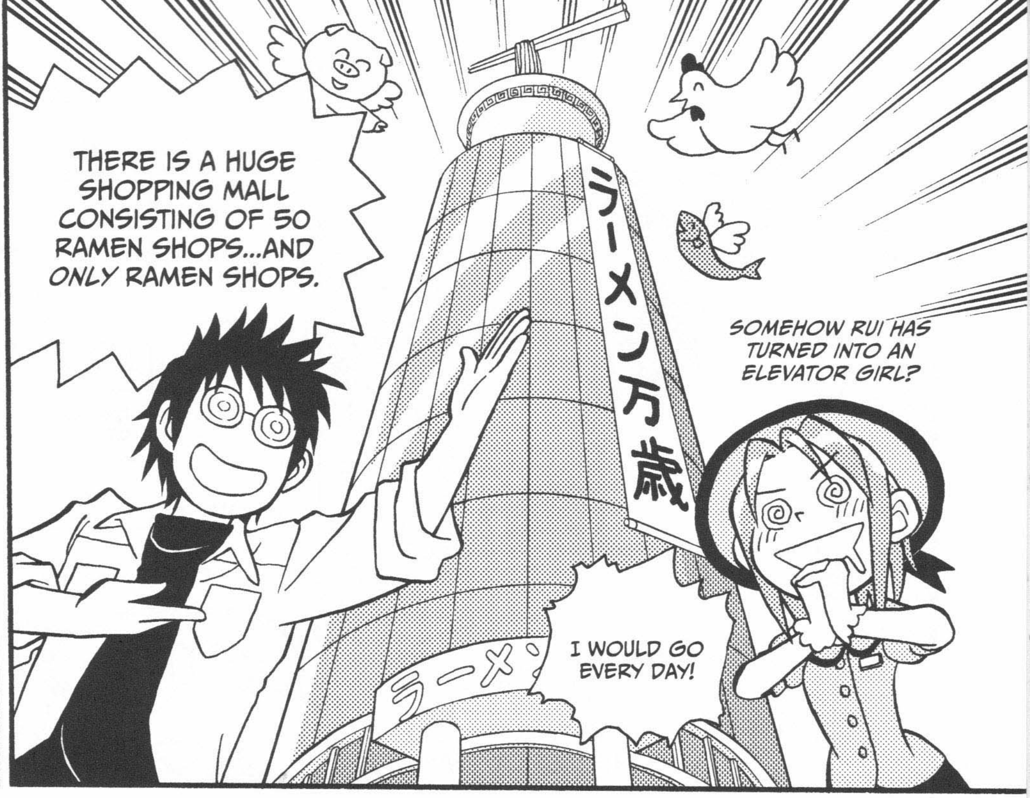


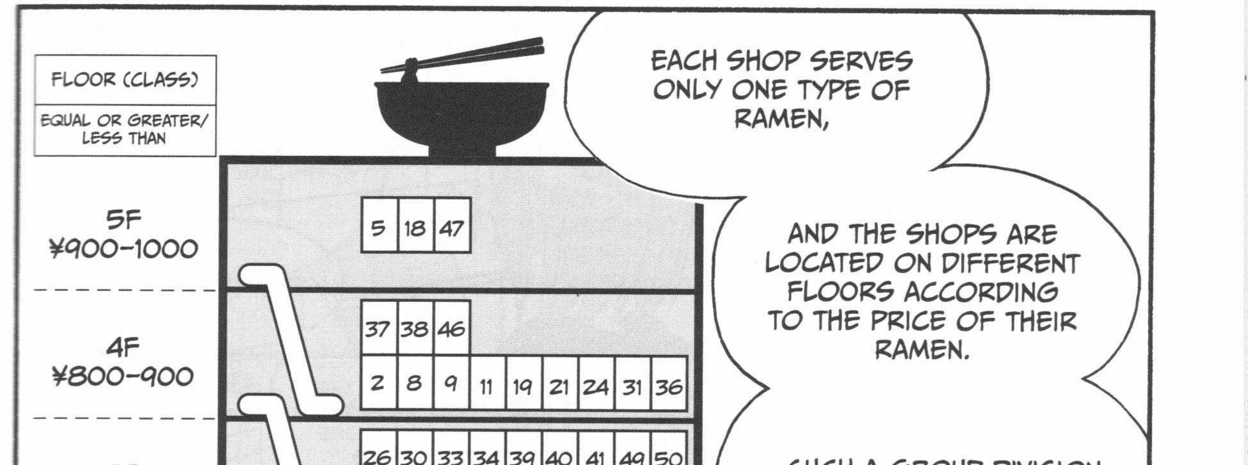


PRICES AT RAMEN SHOPS IN *THE 50 BEST RAMEN SHOPS*

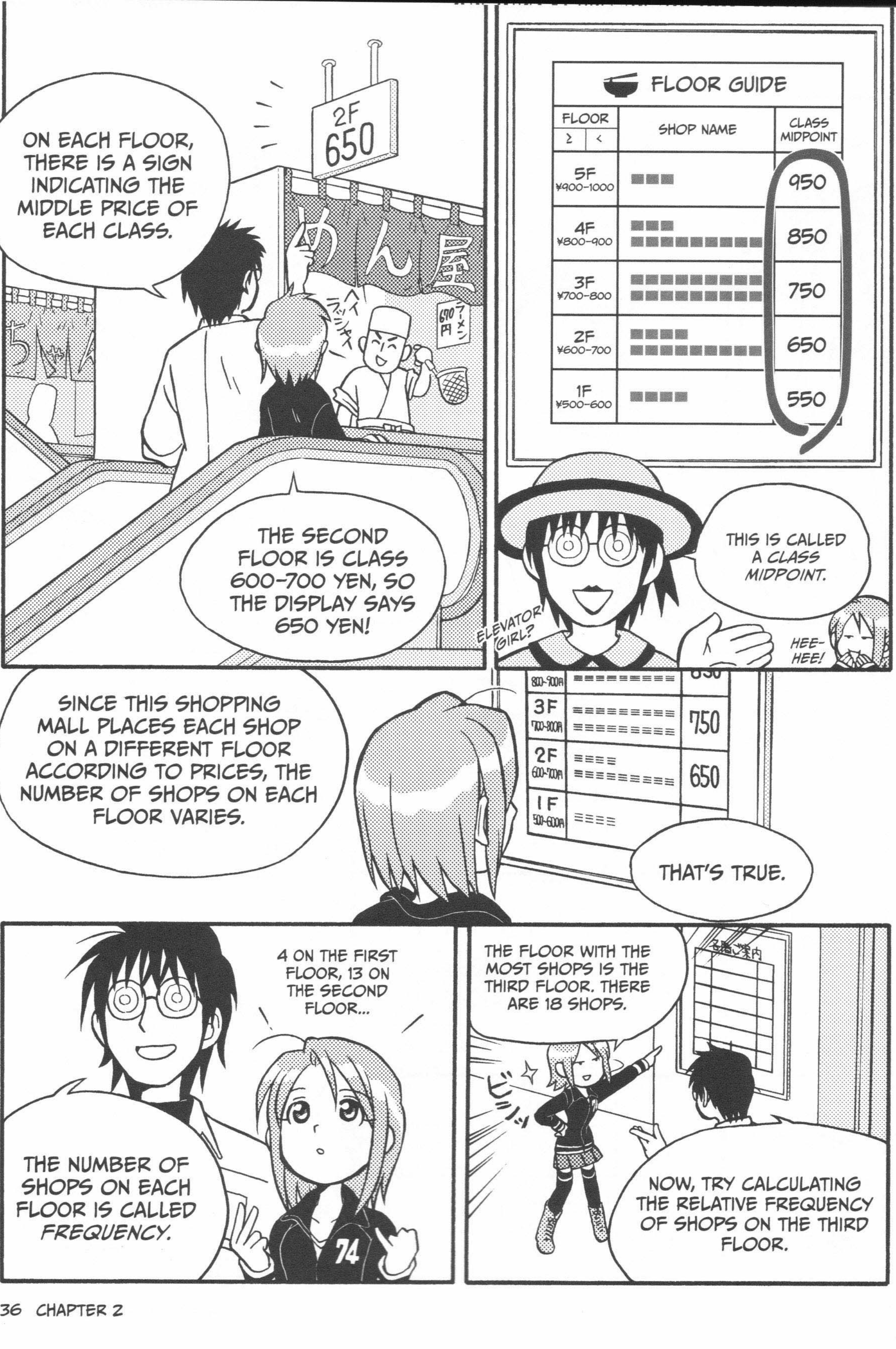
SHOP	PRICE (¥)
1	700
2	850
3	600
4	650
5	980
6	750
7	500
8	890
9	880
10	700
11	890
12	720
13	680
14	650
15	790
16	670
17	680
18	900
19	880
20	720
21	850
22	700
23	780
24	850
25	750
26	780
27	590
28	650
29	580
30	750
31	800
32	550
33	750
34	700
35	600
36	800
37	800
38	880
39	790
40	790
41	780
42	600
43	670
44	680
45	650
46	890
47	930
48	650
49	777
50	700

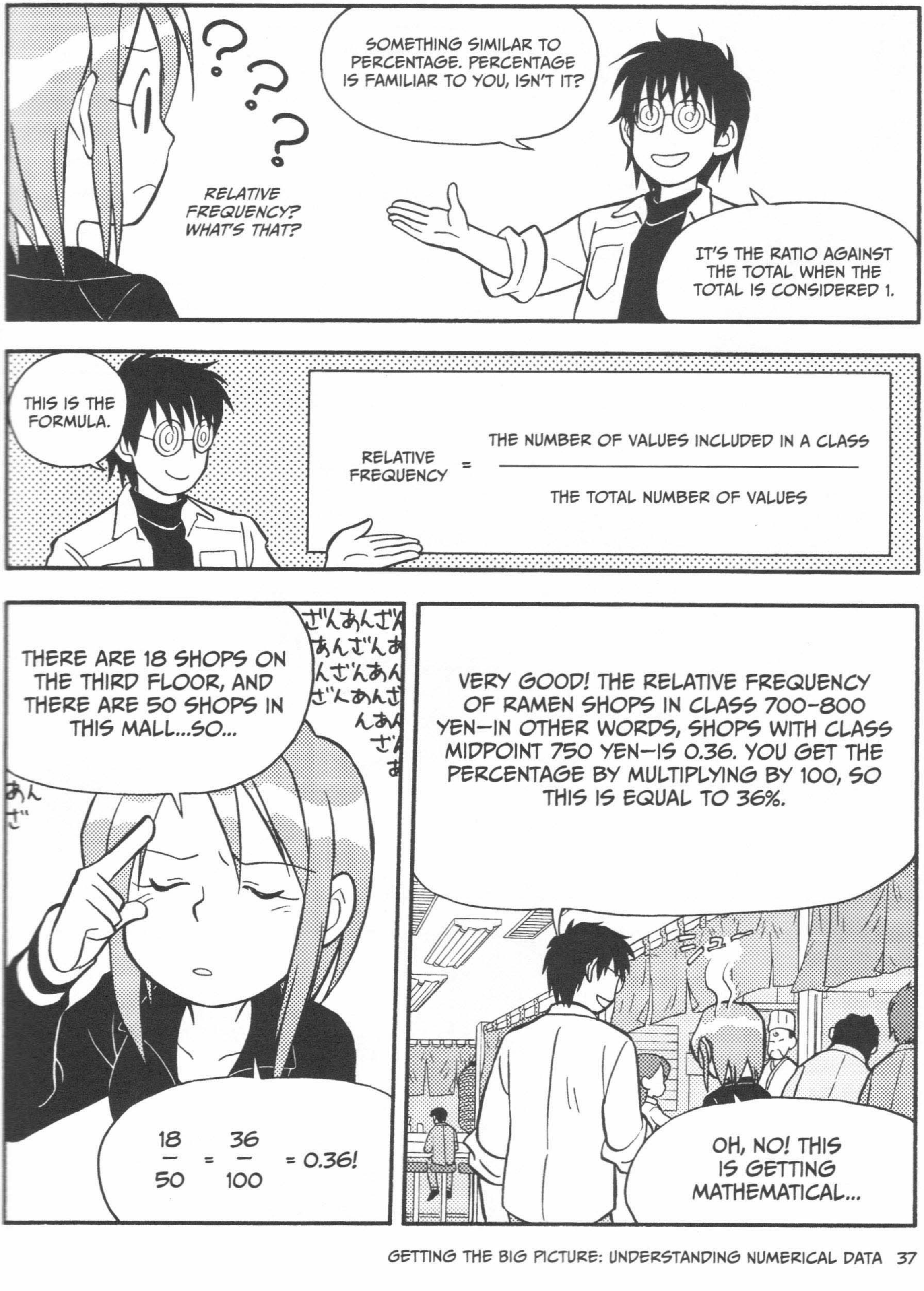

WHAT DO YOU THINK ABOUT THIS TABLE?
WELL...
I FEEL LIKE GOING OUT FOR RAMEN.
ガッ
WHAT?!
ゴオオオオ
YOU *MUST* FEEL SOMETHING OTHER THAN THAT.
I MUST?
BUT I REALLY DON'T...
SHOPS IN THE 50 BEST RAMEN
CE (¥)
SHOP
PRICE
1 700
2 850
3 600
4 650
5 980
6 750
7 500
8 890
9 880
26 780
27 590
28 650
29 580
30 750
31 800
32 550
33
WELL, ONE THING I CAN SEE IS THAT THE PRICES VARY WIDELY.
BUT THIS TABLE IS JUST A BUNCH OF NUMBERS. IT DOESN'T MEAN ANYTHING.
HOW CAN WE MAKE THIS TABLE MORE MEANINGFUL?
HMM.
MAYBE YOU CAN MAKE THIS INTO A GRAPH?
CORRECT!
TO MAKE A GRAPH, WE FIRST MUST MAKE PRICE GROUPS.
?

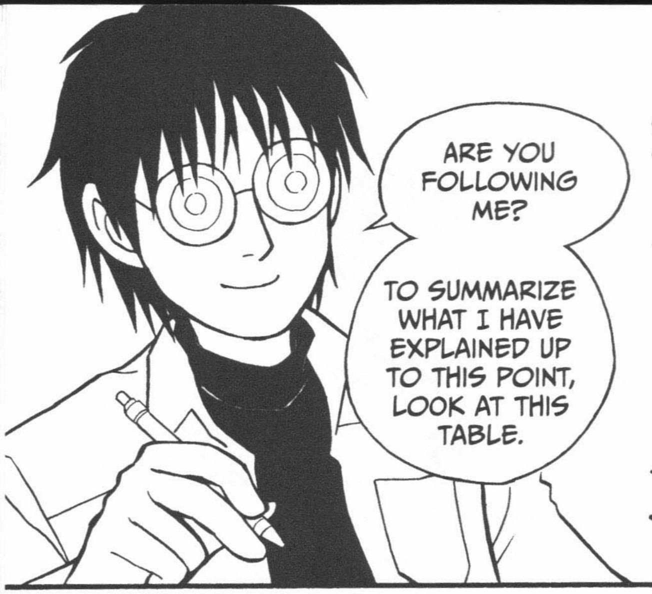




FLOOR (CLASS) EQUAL OR GREATER/ LESS THAN	
5F ¥900–1000	5 18 47
4F ¥800–900	37 38 46 2 8 9 11 19 21 24 31 36
3F ¥700–800	26 30 33 34 39 40 41 49 50 1 6 10 12 15 20 22 23 25
2F ¥600–700	43 44 45 48 3 4 13 14 16 17 28 35 42
1F ¥500–600	7 27 29 32

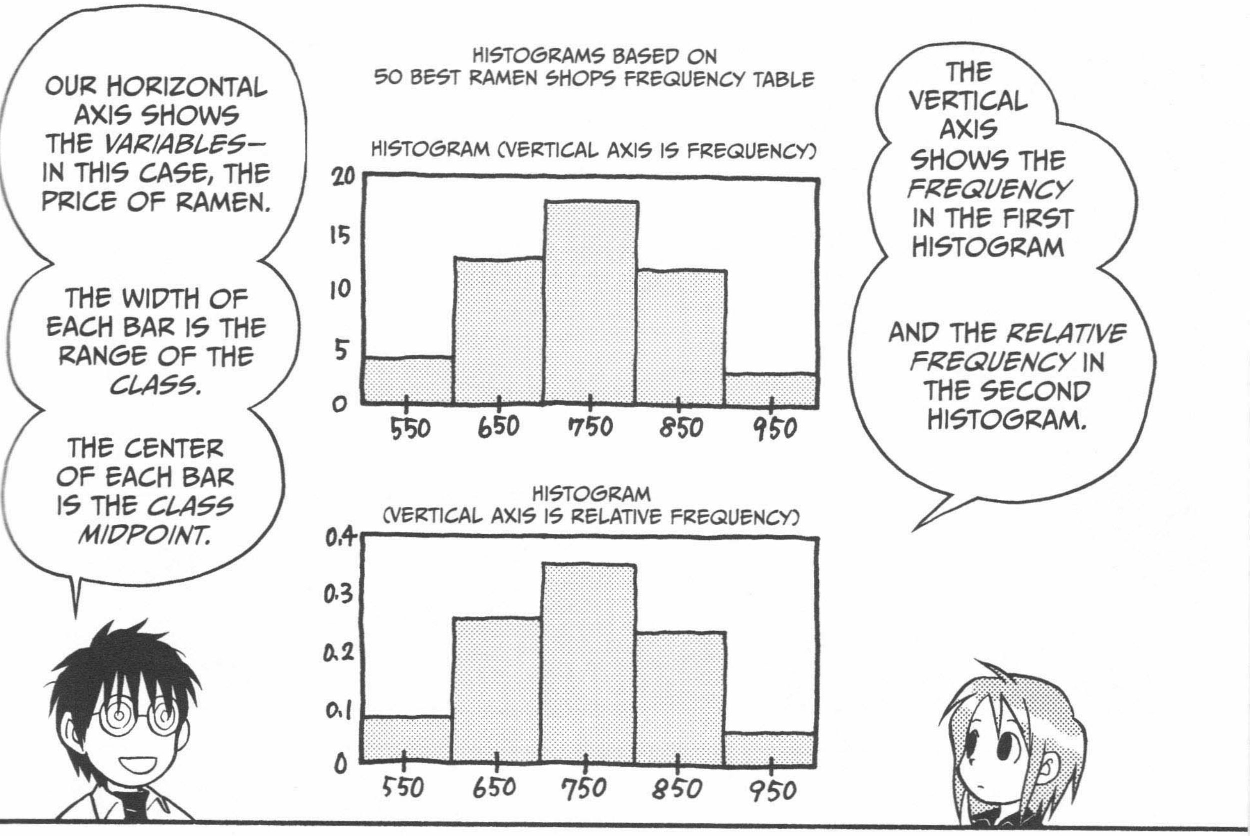

ON EACH FLOOR, THERE IS A SIGN INDICATING THE MIDDLE PRICE OF EACH CLASS.
2F 650
めん屋
ハイラッシャイ
670円 ラーメン
THE SECOND FLOOR IS CLASS 600–700 YEN, SO THE DISPLAY SAYS 650 YEN!
FLOOR GUIDE
FLOOR ≥ <
SHOP NAME
CLASS MIDPOINT
5F ¥900–1000 950
4F ¥800–900 850
3F ¥700–800 750
2F ¥600–700 650
1F ¥500–600 550
THIS IS CALLED A *CLASS MIDPOINT*.
ELEVATOR GIRL?
HEE-HEE!
SINCE THIS SHOPPING MALL PLACES EACH SHOP ON A DIFFERENT FLOOR ACCORDING TO PRICES, THE NUMBER OF SHOPS ON EACH FLOOR VARIES.
800–900円
3F 700–800円 750
2F 600–700円 650
1F 500–600円
THAT'S TRUE.
4 ON THE FIRST FLOOR, 13 ON THE SECOND FLOOR...
THE NUMBER OF SHOPS ON EACH FLOOR IS CALLED *FREQUENCY*.
74
THE FLOOR WITH THE MOST SHOPS IS THE THIRD FLOOR. THERE ARE 18 SHOPS.
各階ご案内
ビシッ
NOW, TRY CALCULATING THE RELATIVE FREQUENCY OF SHOPS ON THE THIRD FLOOR.

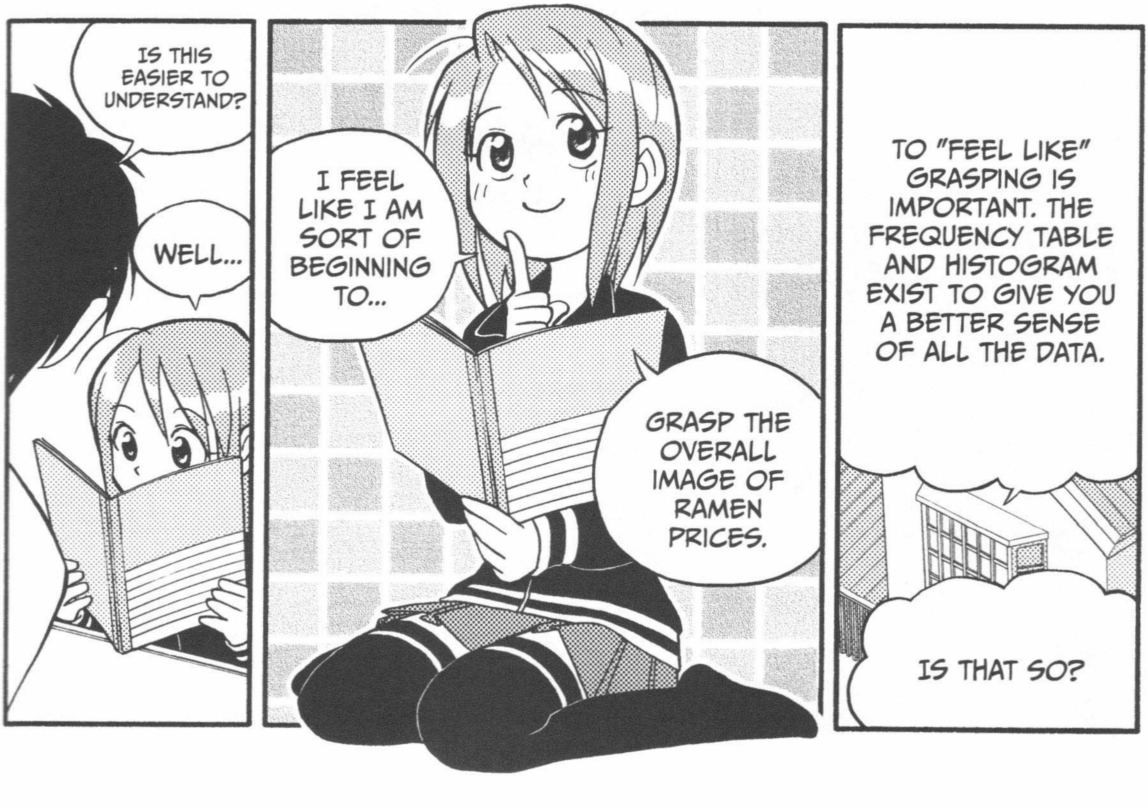

RELATIVE FREQUENCY? WHAT'S THAT?
???
SOMETHING SIMILAR TO PERCENTAGE. PERCENTAGE IS FAMILIAR TO YOU, ISN'T IT?
IT'S THE RATIO AGAINST THE TOTAL WHEN THE TOTAL IS CONSIDERED 1.
THIS IS THE FORMULA.
RELATIVE FREQUENCY = THE NUMBER OF VALUES INCLUDED IN A CLASS / THE TOTAL NUMBER OF VALUES
THERE ARE 18 SHOPS ON THE THIRD FLOOR, AND THERE ARE 50 SHOPS IN THIS MALL...SO...
ざんあんざんあんざんあんざんあんざんあんざんあんざんあんざんあ
あんざ
18/50 = 36/100 = 0.36!
VERY GOOD! THE RELATIVE FREQUENCY OF RAMEN SHOPS IN CLASS 700–800 YEN—IN OTHER WORDS, SHOPS WITH CLASS MIDPOINT 750 YEN—IS 0.36. YOU GET THE PERCENTAGE BY MULTIPLYING BY 100, SO THIS IS EQUAL TO 36%.
ニュー
OH, NO! THIS IS GETTING MATHEMATICAL...



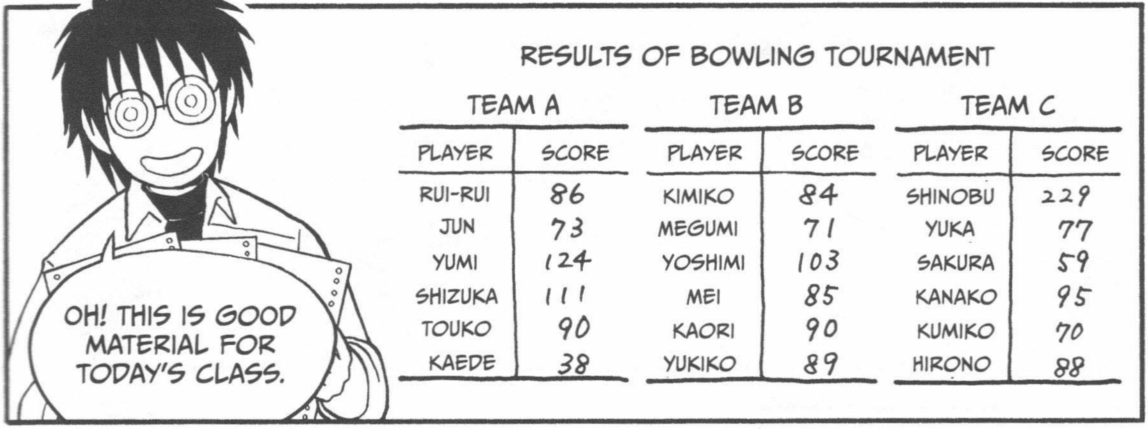
50 BEST RAMEN SHOPS FREQUENCY TABLE

CLASS (EQUAL OR GREATER/ LESS THAN)	CLASS MIDPOINT	FRE-QUENCY	RELATIVE FREQUENCY
500–600	550	4	0.08
600–700	650	13	0.26
700–800	750	18	0.36
800–900	850	12	0.24
900–1000	950	3	0.06
SUM		50	1.00


OUR HORIZONTAL AXIS SHOWS THE *VARIABLES*— IN THIS CASE, THE PRICE OF RAMEN.
THE WIDTH OF EACH BAR IS THE RANGE OF THE *CLASS*.
THE CENTER OF EACH BAR IS THE *CLASS MIDPOINT*.
HISTOGRAMS BASED ON 50 BEST RAMEN SHOPS FREQUENCY TABLE
HISTOGRAM (VERTICAL AXIS IS FREQUENCY)
20
15
10
5
0
550
650
750
850
950
HISTOGRAM (VERTICAL AXIS IS RELATIVE FREQUENCY)
0.4
0.3
0.2
0.1
0
550
650
750
850
950
THE VERTICAL AXIS SHOWS THE *FREQUENCY* IN THE FIRST HISTOGRAM
AND THE *RELATIVE FREQUENCY* IN THE SECOND HISTOGRAM.

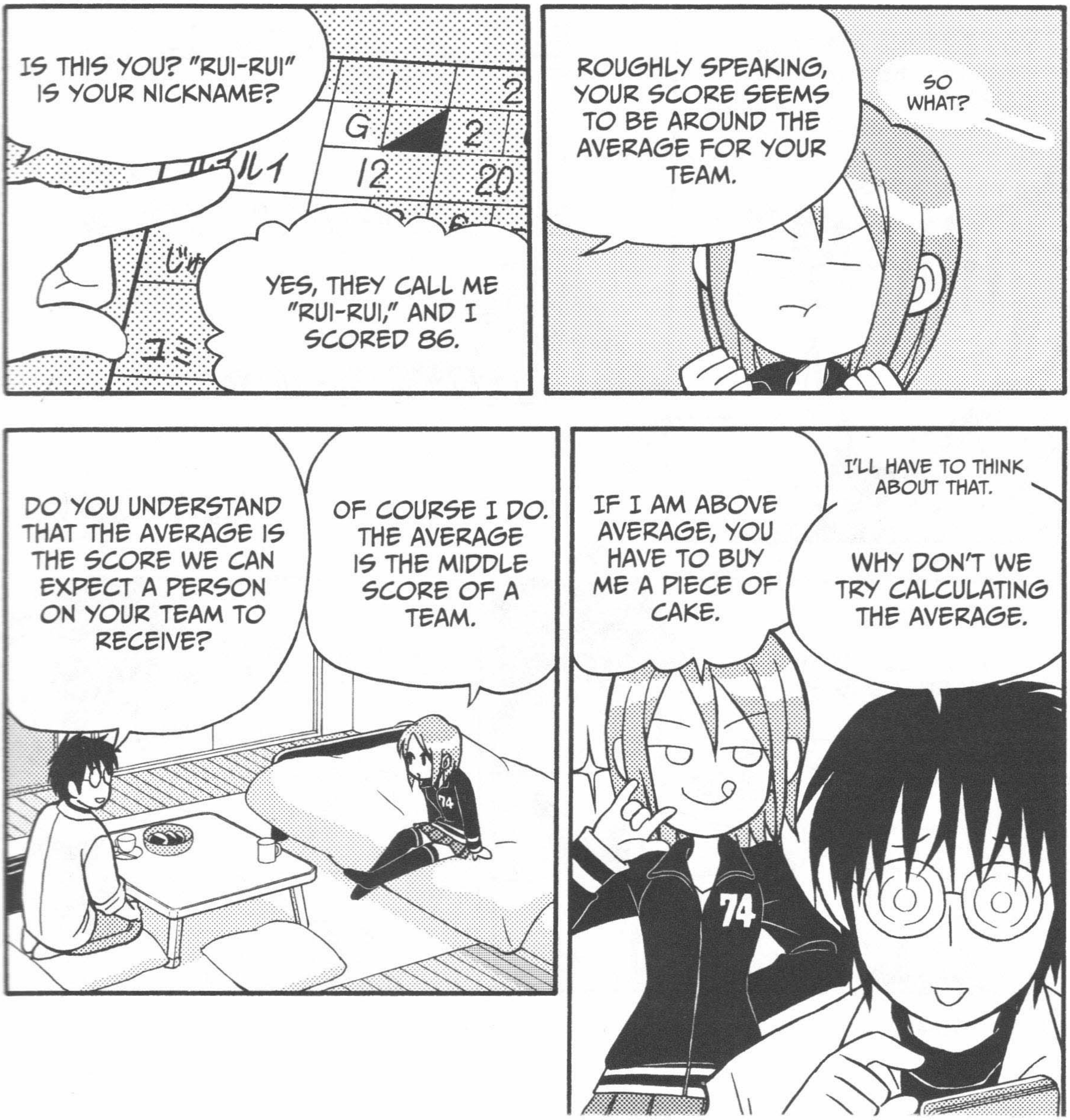

IS THIS EASIER TO UNDERSTAND?
WELL...
I FEEL LIKE I AM SORT OF BEGINNING TO...
GRASP THE OVERALL IMAGE OF RAMEN PRICES.
TO "FEEL LIKE" GRASPING IS IMPORTANT. THE FREQUENCY TABLE AND HISTOGRAM EXIST TO GIVE YOU A BETTER SENSE OF ALL THE DATA.
IS THAT SO?

2. MEAN (AVERAGE)



RESULTS OF BOWLING TOURNAMENT

TEAM A		TEAM B		TEAM C	
PLAYER	SCORE	PLAYER	SCORE	PLAYER	SCORE
RUI-RUI	86	KIMIKO	84	SHINOBU	229
JUN	73	MEGUMI	71	YUKA	77
YUMI	124	YOSHIMI	103	SAKURA	59
SHIZUKA	111	MEI	85	KANAKO	95
TOUKO	90	KAORI	90	KUMIKO	70
KAEDE	38	YUKIKO	89	HIRONO	88



TEAM A

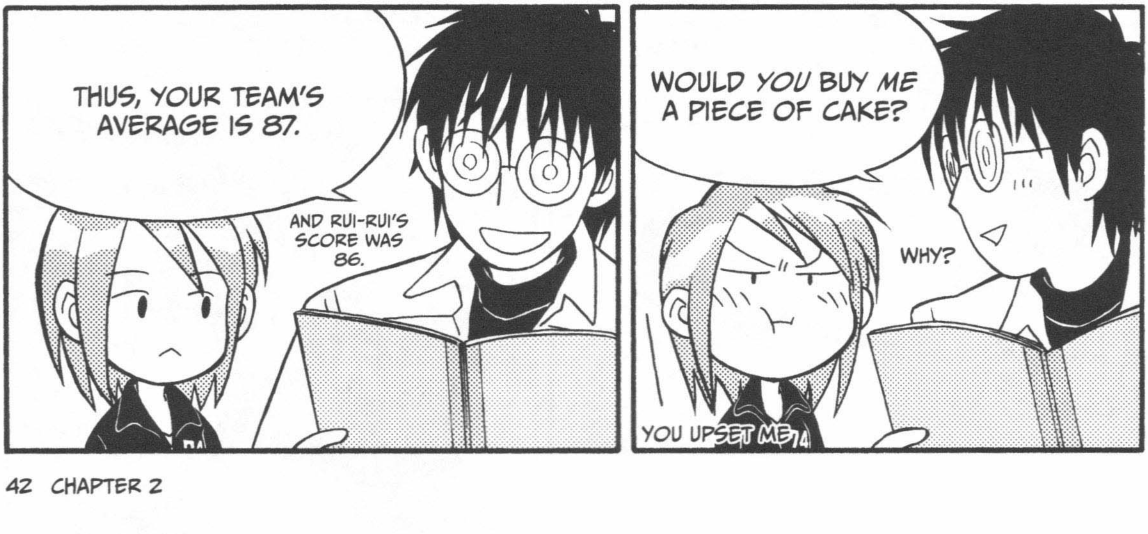
$$\frac{86+73+124+111+90+38}{6}=\frac{522}{6}=87$$

TEAM B

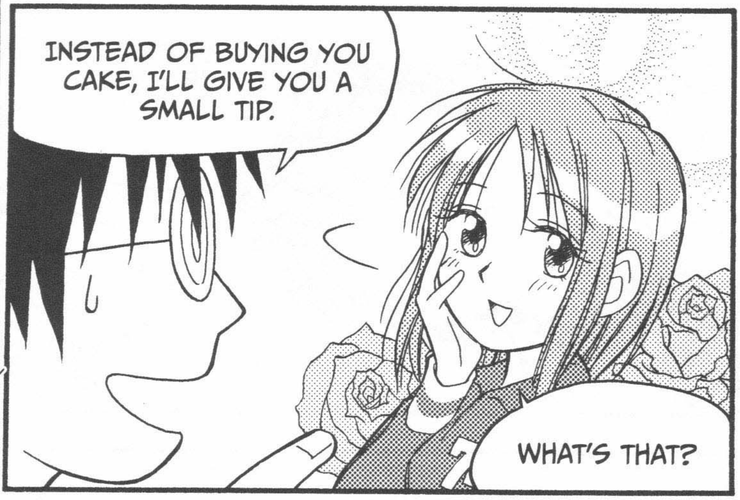
$$\frac{84+71+103+85+90+89}{6}=\frac{522}{6}=87$$

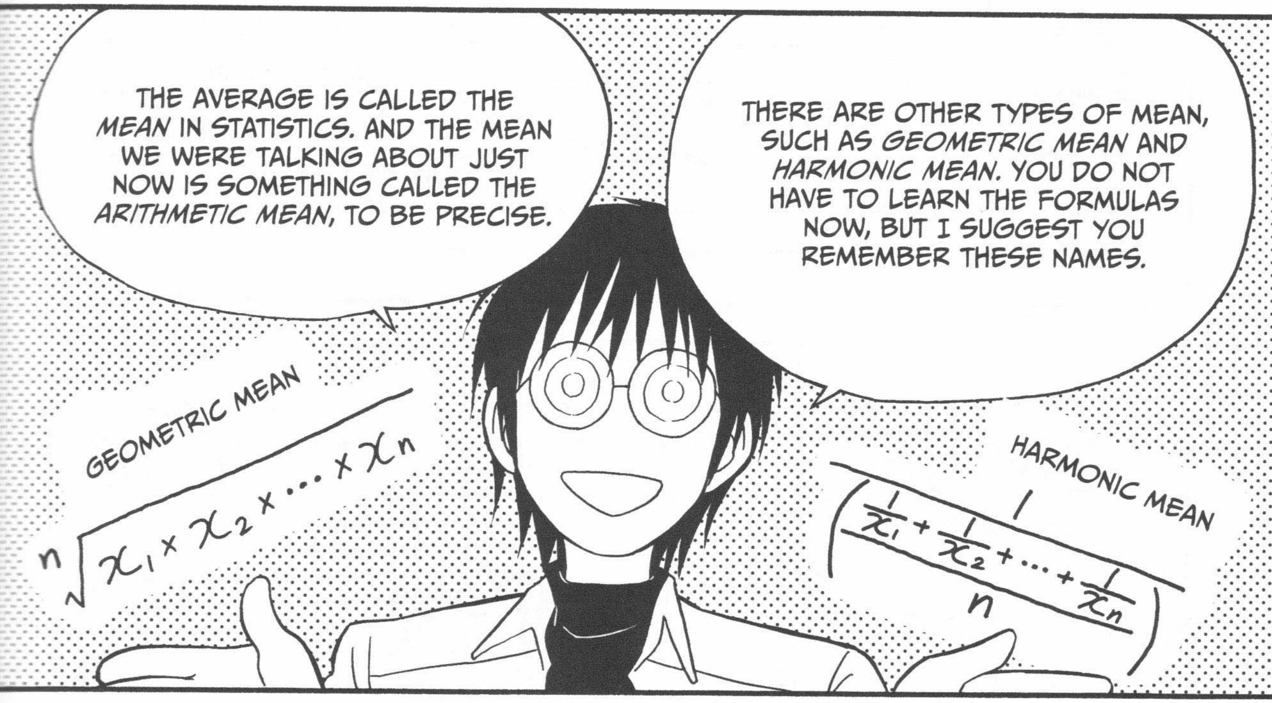
TEAM C

$$\frac{229+77+59+95+70+88}{6}=\frac{618}{6}=103$$



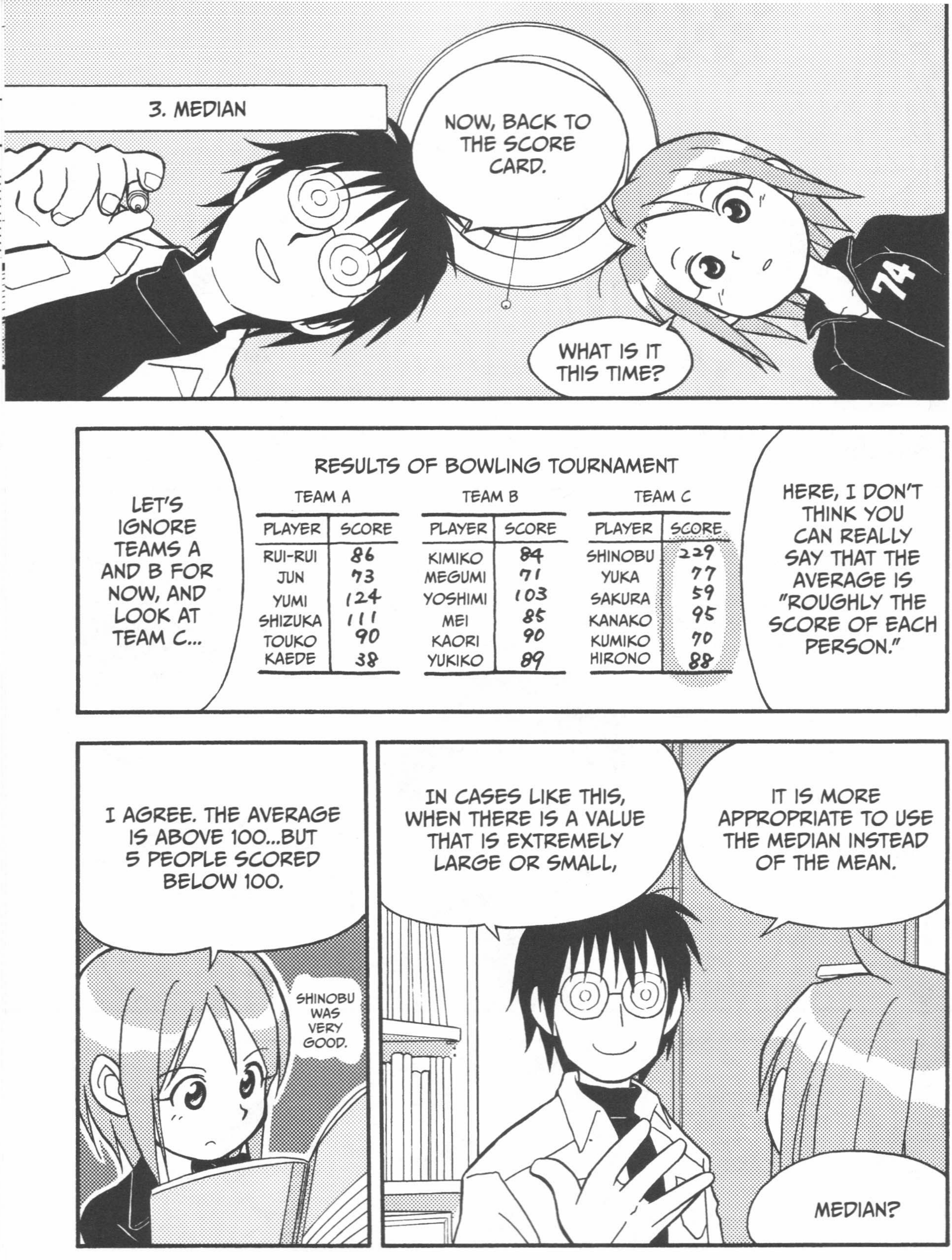

YOU HAVE NO RIGHT TO CALL ME RUI-RUI.
ごめんね
S... SORRY
DON'T BE SO ANGRY...


INSTEAD OF BUYING YOU CAKE, I'LL GIVE YOU A SMALL TIP.
WHAT'S THAT?


THE AVERAGE IS CALLED THE *MEAN* IN STATISTICS. AND THE MEAN WE WERE TALKING ABOUT JUST NOW IS SOMETHING CALLED THE *ARITHMETIC MEAN*, TO BE PRECISE.
THERE ARE OTHER TYPES OF MEAN, SUCH AS *GEOMETRIC MEAN* AND *HARMONIC MEAN*. YOU DO NOT HAVE TO LEARN THE FORMULAS NOW, BUT I SUGGEST YOU REMEMBER THESE NAMES.
GEOMETRIC MEAN
$\sqrt[n]{x_1 \times x_2 \times \cdots \times x_n}$
HARMONIC MEAN
$\frac{1}{\left(\frac{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}{n}\right)}$

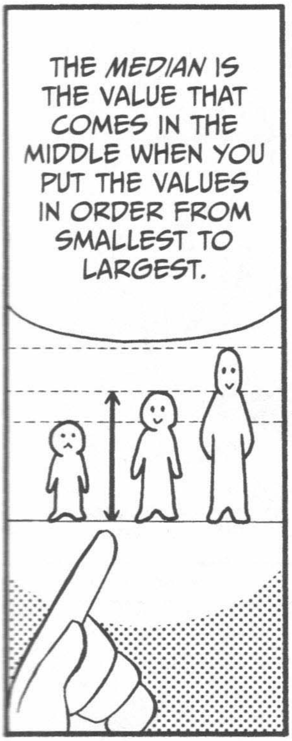

TOO BAD THIS TIP IS NOT AS DELICIOUS AS CAKE.
OH, WELL.

3. MEDIAN



RESULTS OF BOWLING TOURNAMENT

TEAM A		TEAM B		TEAM C	
PLAYER	SCORE	PLAYER	SCORE	PLAYER	SCORE
RUI-RUI	86	KIMIKO	84	SHINOBU	229
JUN	73	MEGUMI	71	YUKA	77
YUMI	124	YOSHIMI	103	SAKURA	59
SHIZUKA	111	MEI	85	KANAKO	95
TOUKO	90	KAORI	90	KUMIKO	70
KAEDE	38	YUKIKO	89	HIRONO	88



TEAM A

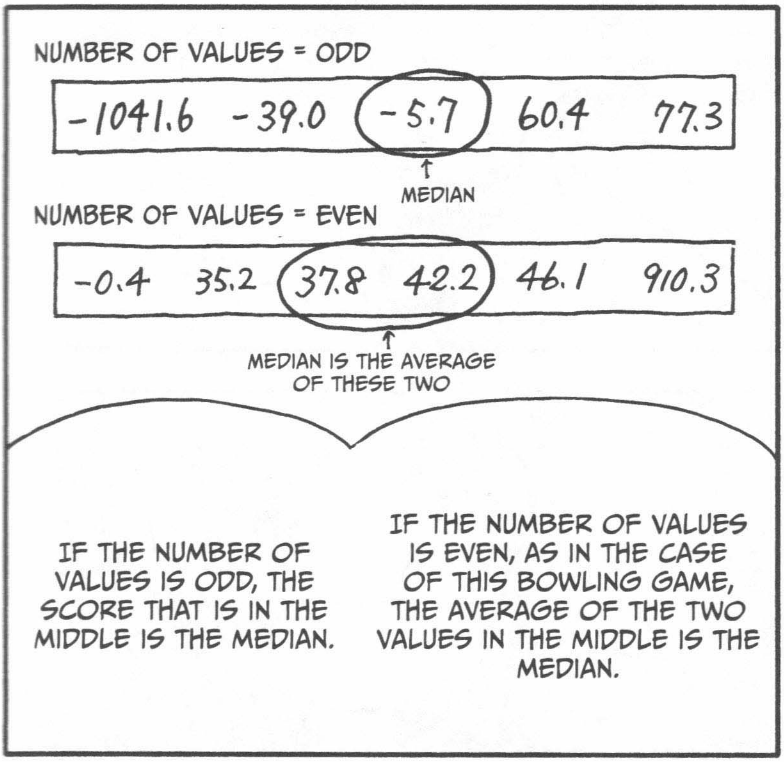
38	73	86	90	111	124

TEAM B

71	84	85	89	90	103

TEAM C

59	70	77	88	95	229



NUMBER OF VALUES = ODD

-1041.6	-39.0	-5.7	60.4	77.3

↑ MEDIAN

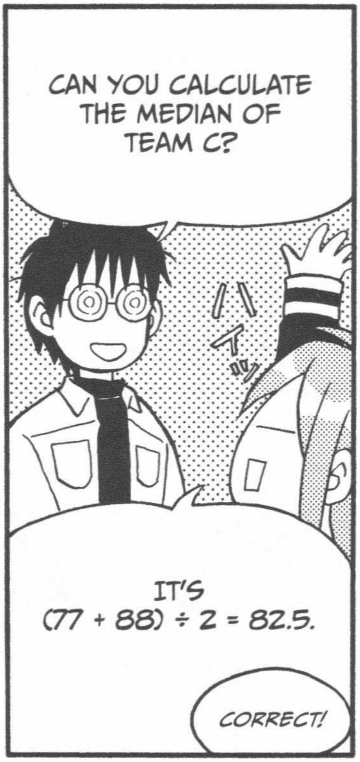
NUMBER OF VALUES = EVEN

-0.4	35.2	37.8	42.2	46.1	910.3

↑ MEDIAN IS THE AVERAGE OF THESE TWO

IF THE NUMBER OF VALUES IS ODD, THE SCORE THAT IS IN THE MIDDLE IS THE MEDIAN.

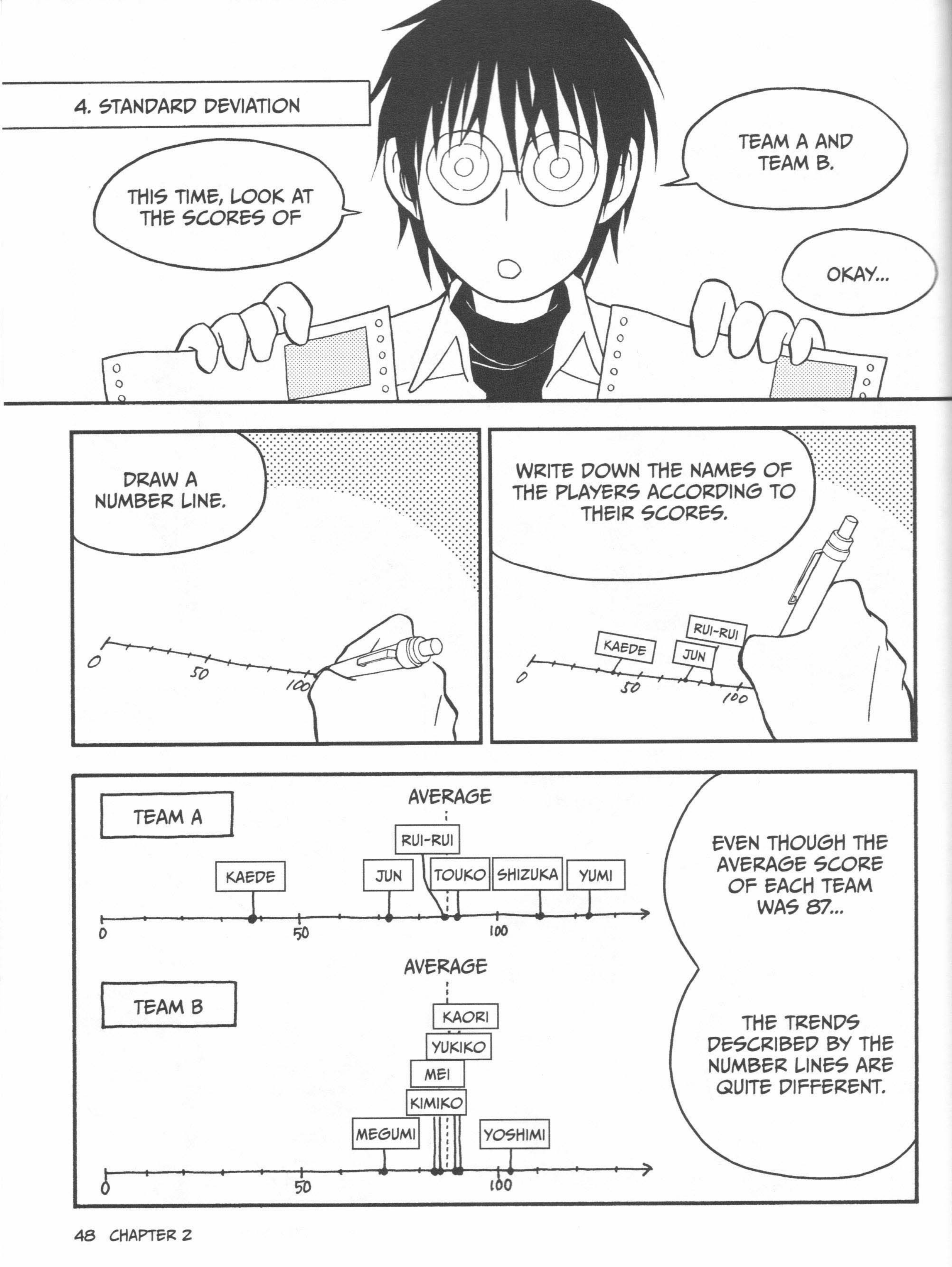
IF THE NUMBER OF VALUES IS EVEN, AS IN THE CASE OF THIS BOWLING GAME, THE AVERAGE OF THE TWO VALUES IN THE MIDDLE IS THE MEDIAN.

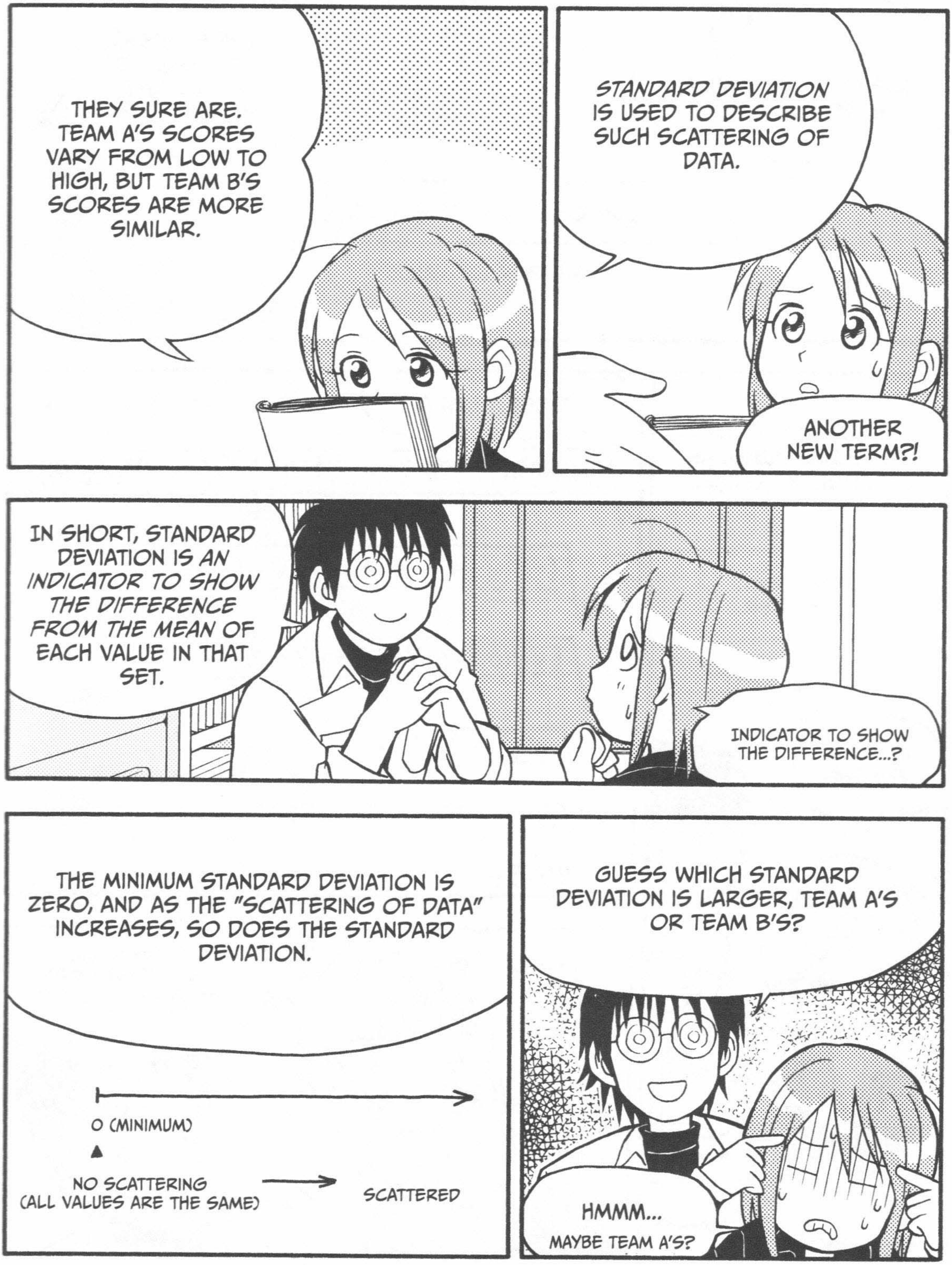


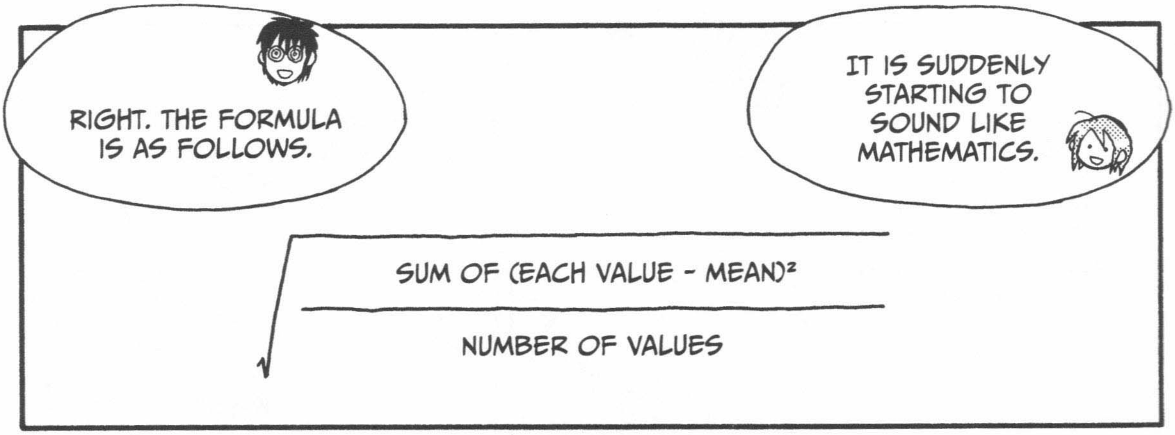
*LESS THAN $100


THE AVERAGE IS HIGH BECAUSE OF SOME MILLIONAIRES.
YOU NEED NOT BE DISAPPOINTED BECAUSE YOUR SAVINGS IS MUCH LESS THAN THE AVERAGE.
IN SUCH CASES, THE MEDIAN IS MUCH CLOSER TO COMMON PEOPLE.
ARE YOU LISTENING?
MILLIONAIRES...
I MUST MARRY A RICH GUY WHOSE SAVINGS IS WAY HIGHER THAN THE MEDIAN!
74
YOU DAMPEN MY SPIRITS...

4. STANDARD DEVIATION




THEY SURE ARE. TEAM A'S SCORES VARY FROM LOW TO HIGH, BUT TEAM B'S SCORES ARE MORE SIMILAR.
STANDARD DEVIATION IS USED TO DESCRIBE SUCH SCATTERING OF DATA.
ANOTHER NEW TERM?!
IN SHORT, STANDARD DEVIATION IS AN *INDICATOR TO SHOW THE DIFFERENCE FROM THE MEAN* OF EACH VALUE IN THAT SET.
INDICATOR TO SHOW THE DIFFERENCE...?
THE MINIMUM STANDARD DEVIATION IS ZERO, AND AS THE "SCATTERING OF DATA" INCREASES, SO DOES THE STANDARD DEVIATION.
0 (MINIMUM)
NO SCATTERING (ALL VALUES ARE THE SAME)
SCATTERED
GUESS WHICH STANDARD DEVIATION IS LARGER, TEAM A'S OR TEAM B'S?
HMMM...
MAYBE TEAM A'S?



$$\sqrt{\frac{\text{SUM OF (EACH VALUE - MEAN)}^2}{\text{NUMBER OF VALUES}}}$$

FIRST, TEAM A.

TEAM A

$$\sqrt{\frac{(86-87)^2+(73-87)^2+(124-87)^2+(111-87)^2+(90-87)^2+(38-87)^2}{6}}$$

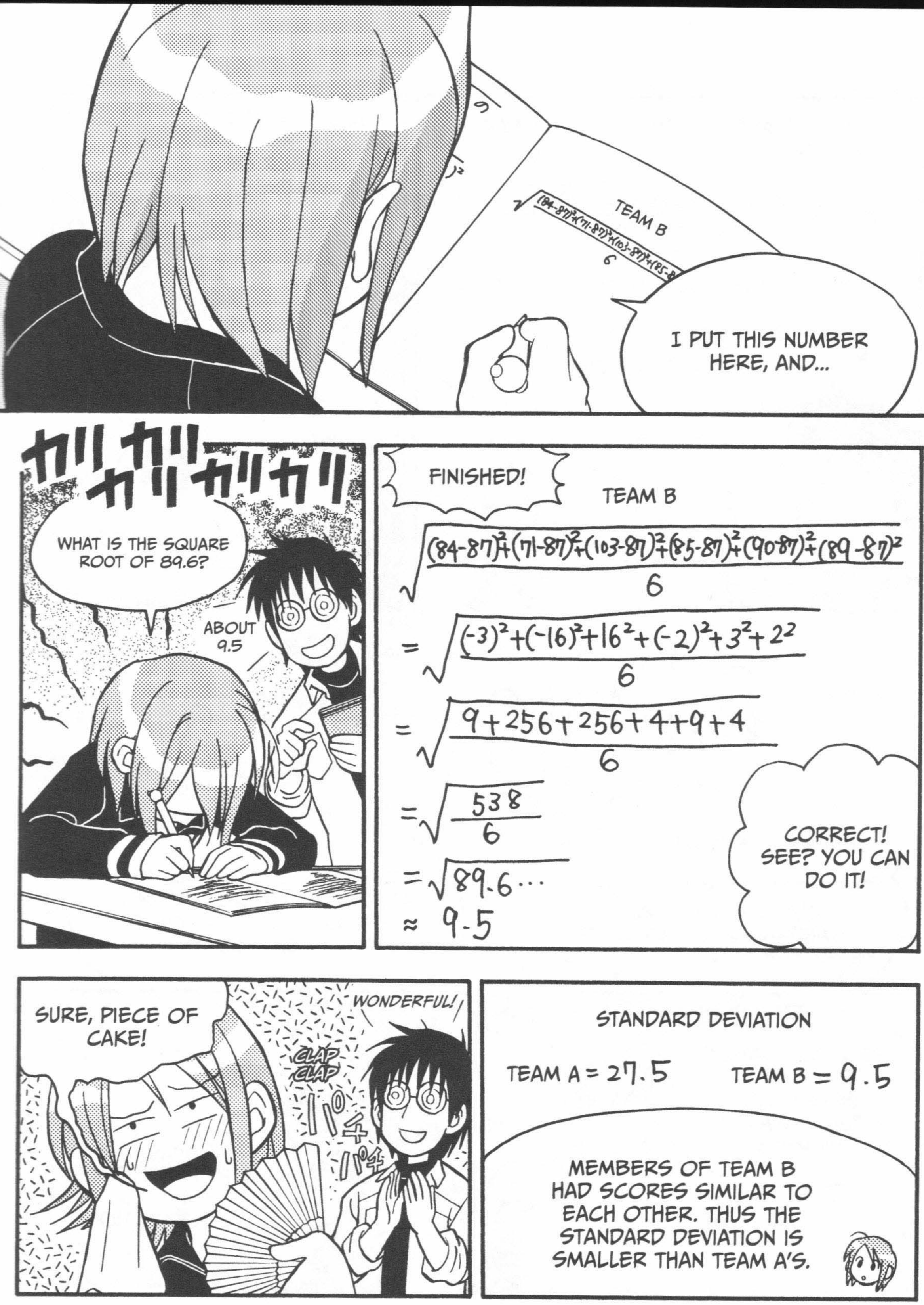
$$=\sqrt{\frac{(-1)^2+(-14)^2+37^2+24^2+3^2+(-49)^2}{6}}$$

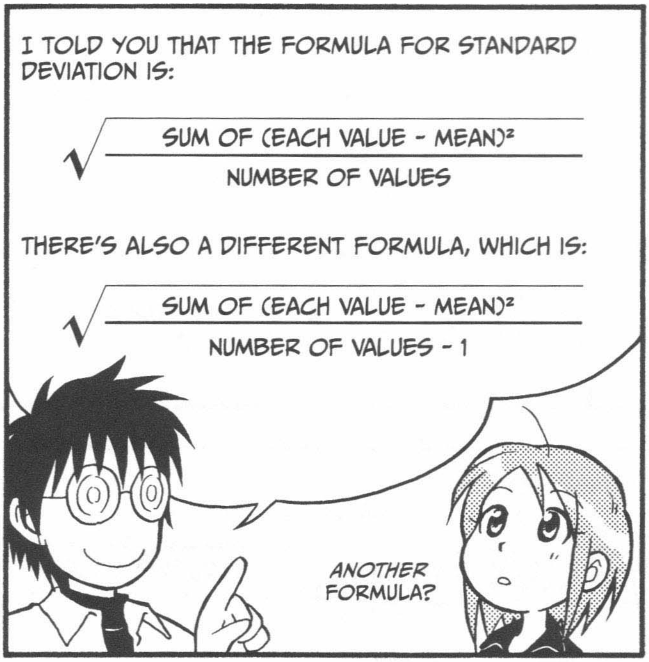
$$=\sqrt{\frac{1+196+1369+576+9+2401}{6}}$$

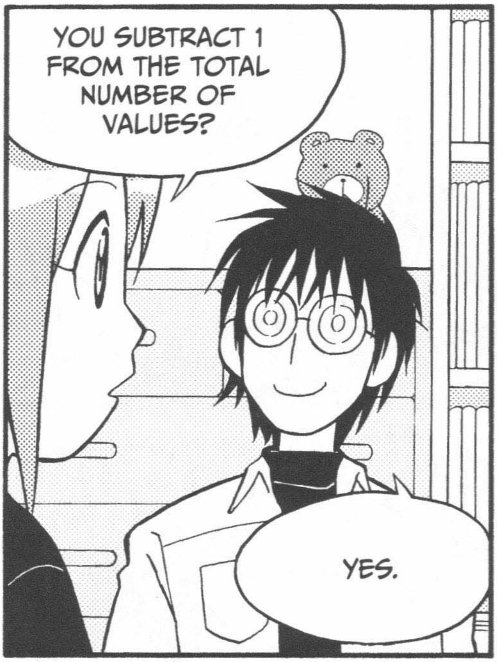
$$=\sqrt{\frac{4552}{6}}$$

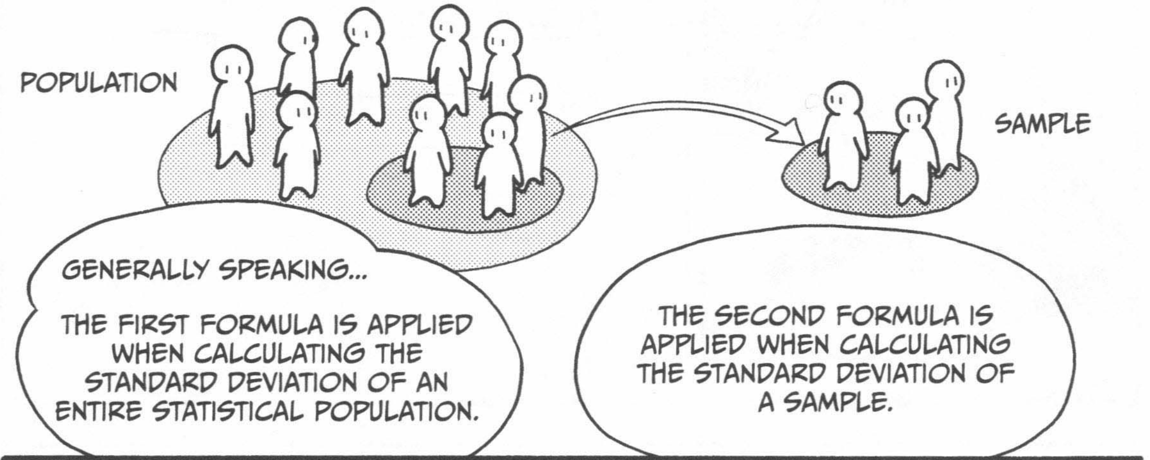
$$=\sqrt{758.6\cdots}$$

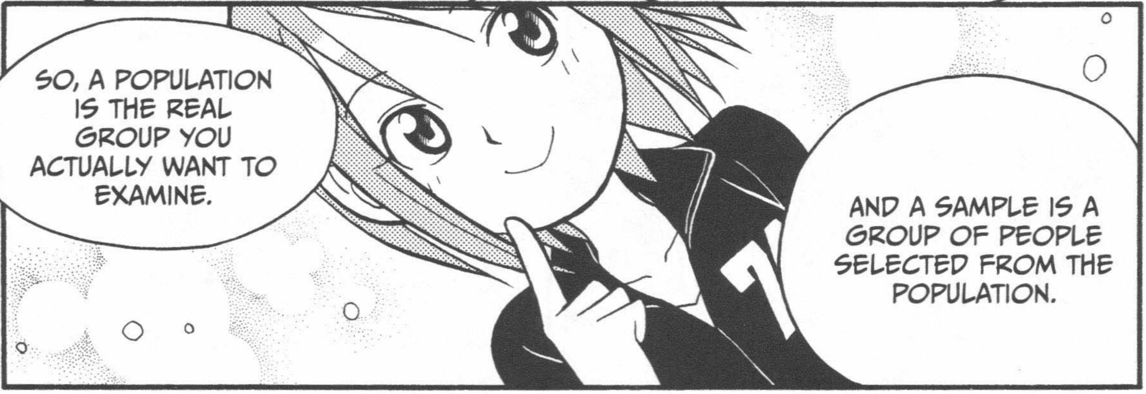
$$\approx 27.5$$

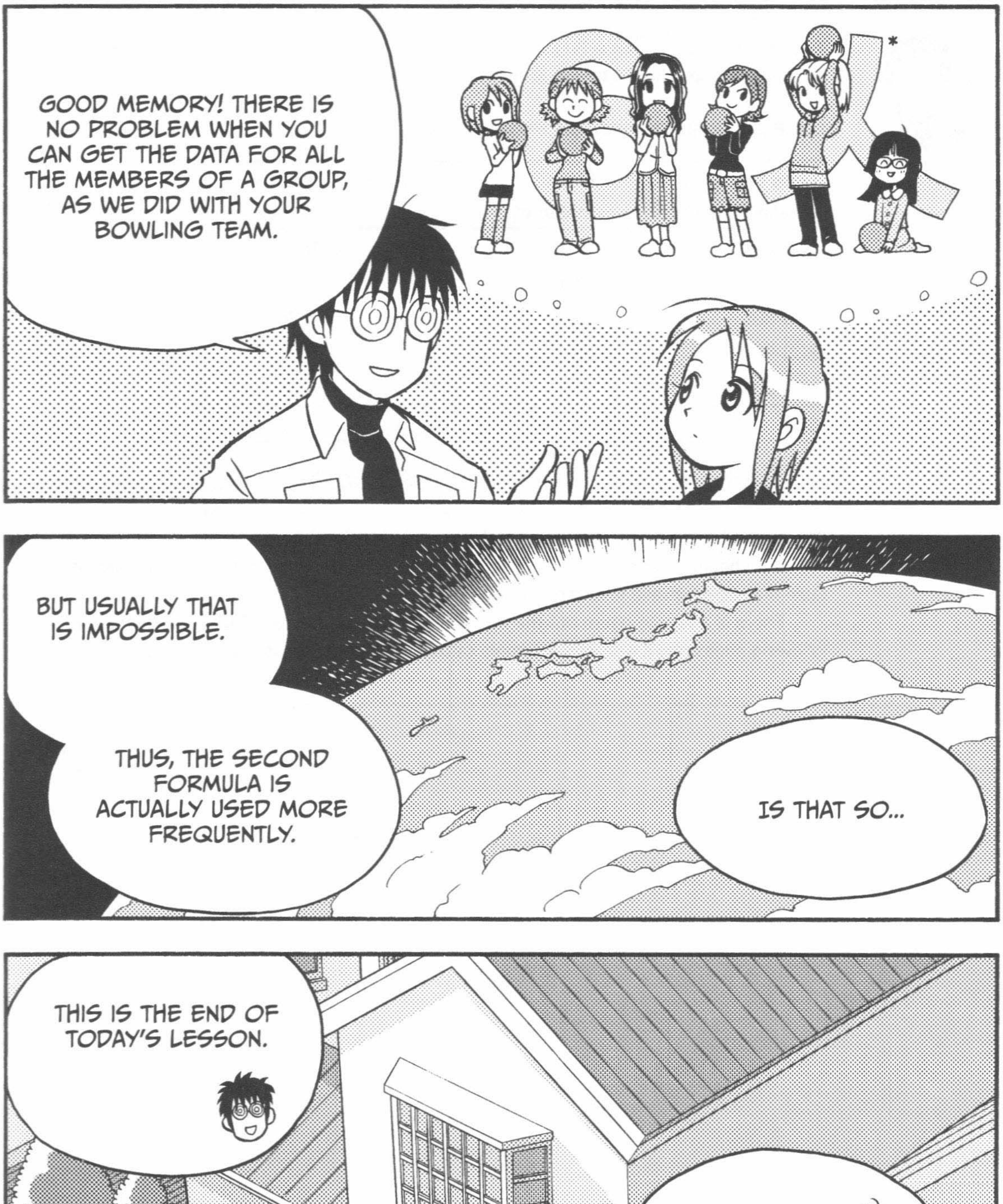

TEAM B
6
I PUT THIS NUMBER HERE, AND...
カリカリ
カリカリカリカリ
WHAT IS THE SQUARE ROOT OF 89.6?
ABOUT 9.5
FINISHED!
TEAM B
$$\sqrt{\frac{(84-87)^2+(71-87)^2+(103-87)^2+(85-87)^2+(90-87)^2+(89-87)^2}{6}}$$
$$=\sqrt{\frac{(-3)^2+(-16)^2+16^2+(-2)^2+3^2+2^2}{6}}$$
$$=\sqrt{\frac{9+256+256+4+9+4}{6}}$$
$$=\sqrt{\frac{538}{6}}$$
$$=\sqrt{89.6\cdots}$$
$$\approx 9.5$$
CORRECT! SEE? YOU CAN DO IT!
SURE, PIECE OF CAKE!
WONDERFUL!
CLAP CLAP
パチ パチ
STANDARD DEVIATION
TEAM A = 27.5
TEAM B = 9.5
MEMBERS OF TEAM B HAD SCORES SIMILAR TO EACH OTHER. THUS THE STANDARD DEVIATION IS SMALLER THAN TEAM A'S.


I TOLD YOU THAT THE FORMULA FOR STANDARD DEVIATION IS:
$\sqrt{\frac{\text{SUM OF (EACH VALUE - MEAN)}^2}{\text{NUMBER OF VALUES}}}$
THERE'S ALSO A DIFFERENT FORMULA, WHICH IS:
$\sqrt{\frac{\text{SUM OF (EACH VALUE - MEAN)}^2}{\text{NUMBER OF VALUES - 1}}}$
ANOTHER FORMULA?


YOU SUBTRACT 1 FROM THE TOTAL NUMBER OF VALUES?
YES.


POPULATION
SAMPLE
GENERALLY SPEAKING... THE FIRST FORMULA IS APPLIED WHEN CALCULATING THE STANDARD DEVIATION OF AN ENTIRE STATISTICAL POPULATION.
THE SECOND FORMULA IS APPLIED WHEN CALCULATING THE STANDARD DEVIATION OF A SAMPLE.


SO, A POPULATION IS THE REAL GROUP YOU ACTUALLY WANT TO EXAMINE.
AND A SAMPLE IS A GROUP OF PEOPLE SELECTED FROM THE POPULATION.


GOOD MEMORY! THERE IS NO PROBLEM WHEN YOU CAN GET THE DATA FOR ALL THE MEMBERS OF A GROUP, AS WE DID WITH YOUR BOWLING TEAM.
BUT USUALLY THAT IS IMPOSSIBLE.
THUS, THE SECOND FORMULA IS ACTUALLY USED MORE FREQUENTLY.
IS THAT SO...
THIS IS THE END OF TODAY'S LESSON.
THANK YOU, MR. YAMAMOTO.

*6 MEMBERS

5. THE RANGE OF CLASS OF A FREQUENCY TABLE

If you felt that something was unclear in "Frequency Distribution Tables and Histograms" on page 32, take another look here at the table introduced on page 38.

TABLE 2-1: 50 BEST RAMEN SHOPS FREQUENCY TABLE

Class (equal or greater/less than)	Class midpoint	Frequency	Relative frequency
500–600	550	4	0.08
600–700	650	13	0.26
700–800	750	18	0.36
800–900	850	12	0.24
900–1000	950	3	0.06
Sum		50	1.00

As you can see, the range of class in this table is 100. The range was not determined according to any kind of mathematical standard—I set the range subjectively. Determining the range of class is up to the person who is analyzing the data.

But shouldn't there be a way to set the range of class mathematically? A frequency table may seem invalid if its range is determined subjectively.

There is a way to figure out the range of class mathematically. This is explained on the following pages. You'll also find a sample calculation using the data in Table 2-1.

Step 1

Calculate the number of classes using the Sturges' Rule below:

$$1 + \frac{\log_{10}(\text{number of values})}{\log_{10}2}$$

$$1 + \frac{\log_{10}50}{\log_{10}2} = 1 + 5.6438... = 6.6438... \approx 7$$

Step 2

Calculate the range of class using the formula below:

$$\frac{(\text{the maximum value}) - (\text{the minimum value})}{\text{the number of classes calculated from the Sturges' Rule}}$$

$$\frac{980 - 500}{7} = \frac{480}{7} = 68.5714... \approx 69$$

Below is a frequency chart organized according to the range of class as calculated by the formula in step 2.

TABLE 2-2: 50 BEST RAMEN SHOPS FREQUENCY TABLE (RANGE OF CLASS DETERMINED MATHEMATICALLY)

Class (equal or greater/less than)	**Class midpoint**	**Frequency**	**Relative frequency**
500–569	534.5	2	0.04
569–638	603.5	5	0.10
638–707	672.5	15	0.30
707–776	741.5	6	0.12
776–845	810.5	10	0.20
845–914	879.5	10	0.20
914–983	948.5	2	0.04
Sum		50	1.00

What do you think of this? Does this table seem even less convincing compared to Table 2-1? And why is the interval 69 yen?

If you try to explain to people that "this was calculated by a formula called the Sturges' Rule," they will only get mad and say, "Who cares about Stur . . . whatever! Why did you set the interval to a weird amount like 69 yen?"

To summarize, some people may hesitate to set the range of class subjectively. However, as the table above indicates, determining the range of class with the Sturges' Rule does not necessarily provide a convincing table. A frequency table is, after all, a tool to help you visualize data. The analyst should set the range of class to any amount he or she thinks is appropriate.

6. ESTIMATION THEORY AND DESCRIPTIVE STATISTICS

In the prologue, we explain that statistics can make an estimate about the situation of the population based on information collected from samples. To tell the truth, this explanation is not necessarily correct.

Statistics can be roughly classified into two categories: estimation theory and descriptive statistics. The one introduced in the prologue is the former. What, then, is descriptive statistics? It is a kind of a statistics that aims to describe the status of a group simply and clearly by organizing data. Descriptive statistics regards the group as the population.

Perhaps this explanation of descriptive statistics is abstract and difficult to understand. Here is an example to help clarify things. Remember when I figured out the mean and standard deviation of Rui's bowling team? This was not because I was trying to estimate the status of a population from the information collected from Rui's team. I calculated the mean and standard deviation purely because I wanted to describe the status of Rui's team simply. That kind of statistics is descriptive statistics.

EXERCISE AND ANSWER

EXERCISE

The table below is a record of a high school girls' 100m track race.

Runner	100m track race (seconds)
Ms. A	16.3
Ms. B	22.4
Ms. C	18.5
Ms. D	18.7
Ms. E	20.1

1. What is the average?
2. What is the median?
3. What is the standard deviation?

ANSWER

1. The arithmetic mean is $\dfrac{16.3 + 22.4 + 18.5 + 18.7 + 20.1}{5} = \dfrac{96}{5} = 19.2$

2. The median is 18.7. $16.3 \quad 18.5 \quad \boxed{18.7} \quad 20.1 \quad 22.4$

3. The standard deviation is

$$\sqrt{\frac{(16.3 - 19.2)^2 + (22.4 - 19.2)^2 + (18.5 - 19.2)^2 + (18.7 - 19.2)^2 + (20.1 - 19.2)^2}{5}}$$

$$= \sqrt{\frac{(-2.9)^2 + 3.2^2 + (-0.7)^2 + (-0.5)^2 + 0.9^2}{5}}$$

$$= \sqrt{\frac{8.41 + 10.24 + 0.49 + 0.25 + 0.81}{5}}$$

$$= \sqrt{\frac{20.2}{5}}$$

$$= \sqrt{4.04}$$

$$\approx 2.01$$

SUMMARY

- To visualize the big picture of the data intuitively, create a frequency table or draw a histogram.
- When making a frequency table, the range of class may be determined by the Sturges' Rule.
- To visualize the data mathematically, calculate the arithmetic mean, median, and standard deviation.
- When there is an extremely large or small value in the data set, it is more appropriate to use the median than the arithmetic mean.
- *Standard deviation* is an index to describe "the size of scattering" of the data.

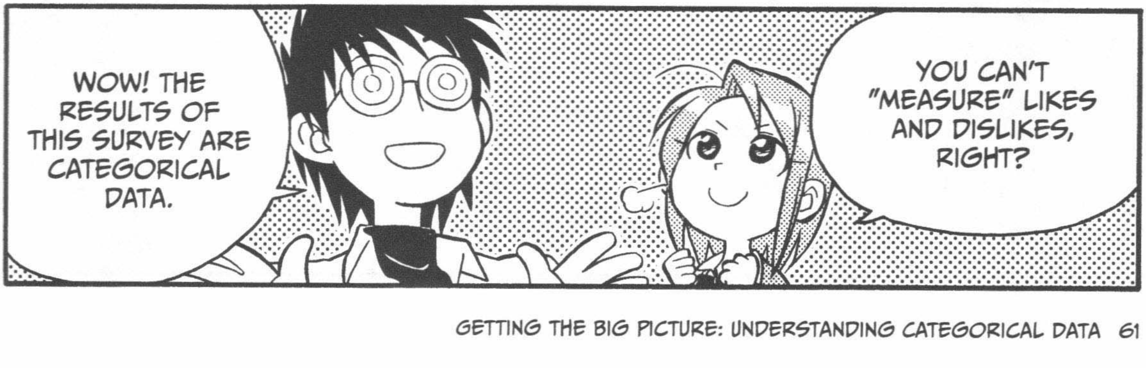
3

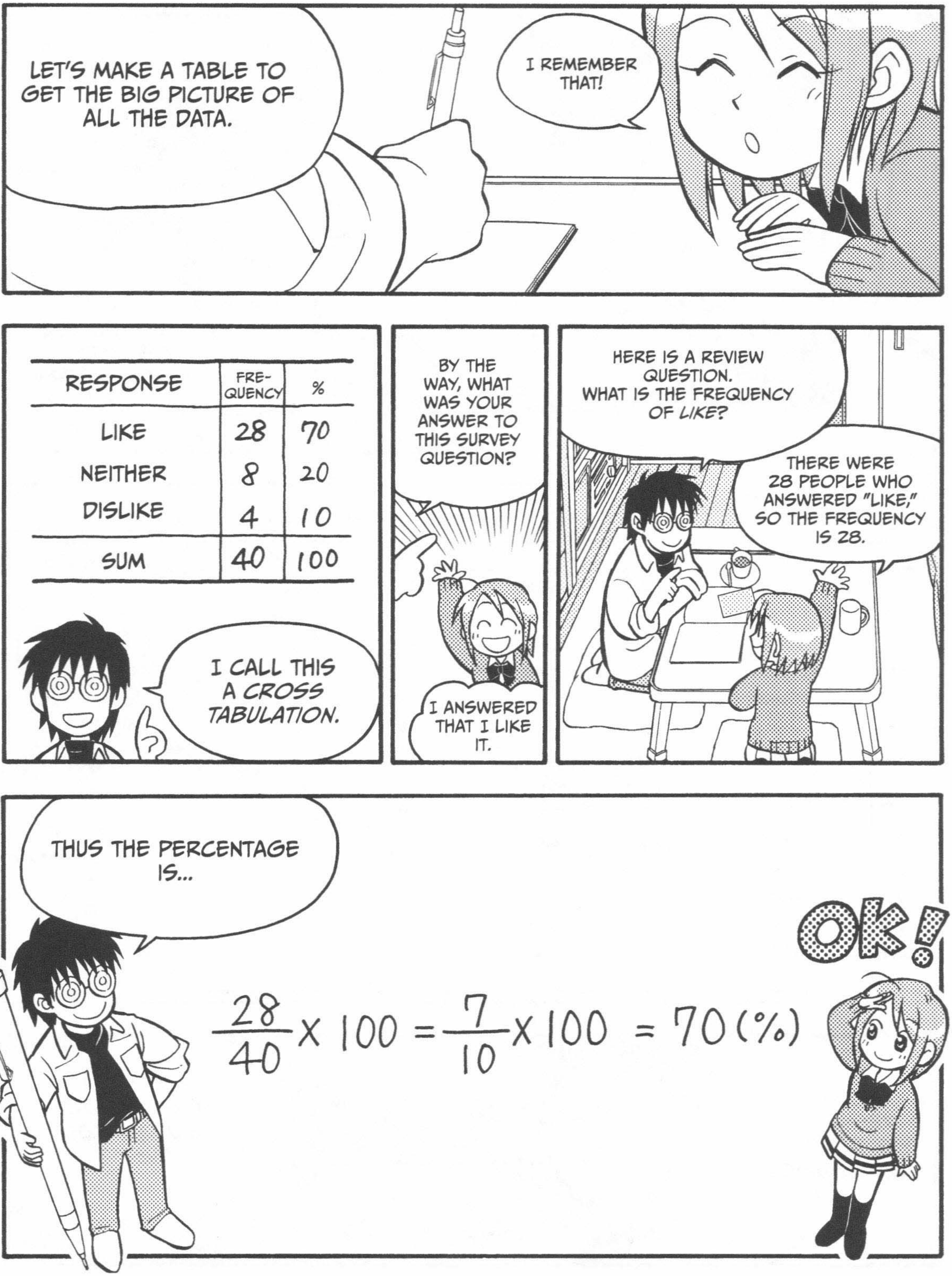
GETTING THE BIG PICTURE: UNDERSTANDING CATEGORICAL DATA

1. CROSS TABULATIONS

DO YOU LIKE OR DISLIKE THE NEW UNIFORM DESIGN?

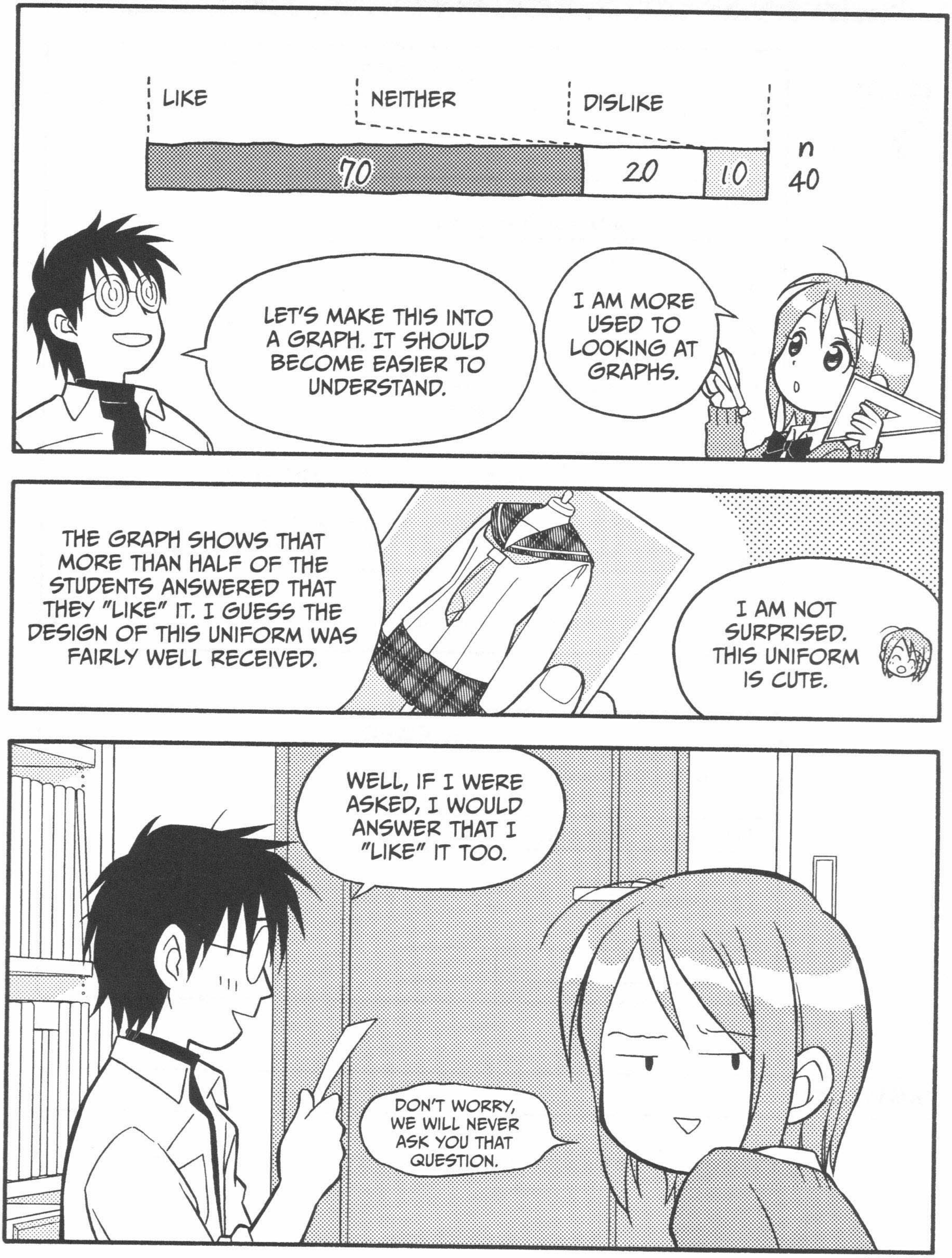
	RESPONSE		RESPONSE		RESPONSE
1	LIKE	16	NEITHER	31	NEITHER
2	NEITHER	17	LIKE	32	NEITHER
3	LIKE	18	LIKE	33	LIKE
4	NEITHER	19	LIKE	34	DISLIKE
5	DISLIKE	20	LIKE	35	LIKE
6	LIKE	21	LIKE	36	LIKE
7	LIKE	22	LIKE	37	LIKE
8	LIKE	23	DISLIKE	38	LIKE
9	LIKE	24	NEITHER	39	NEITHER
10	LIKE	25	LIKE	40	LIKE
11	LIKE	26	LIKE		
12	LIKE	27	DISLIKE		
13	NEITHER	28	LIKE		
14	LIKE	29	LIKE		
15	LIKE	30	LIKE		





RESPONSE	FRE-QUENCY	%
LIKE	28	70
NEITHER	8	20
DISLIKE	4	10
SUM	40	100

$$\frac{28}{40} \times 100 = \frac{7}{10} \times 100 = 70 (\%)$$


LIKE
NEITHER
DISLIKE
70
20
10
n
40
LET'S MAKE THIS INTO A GRAPH. IT SHOULD BECOME EASIER TO UNDERSTAND.
I AM MORE USED TO LOOKING AT GRAPHS.
THE GRAPH SHOWS THAT MORE THAN HALF OF THE STUDENTS ANSWERED THAT THEY "LIKE" IT. I GUESS THE DESIGN OF THIS UNIFORM WAS FAIRLY WELL RECEIVED.
I AM NOT SURPRISED. THIS UNIFORM IS CUTE.
WELL, IF I WERE ASKED, I WOULD ANSWER THAT I "LIKE" IT TOO.
DON'T WORRY, WE WILL NEVER ASK YOU THAT QUESTION.

EXERCISE AND ANSWER

EXERCISE

A newspaper took a survey on political party A, which hopes to win the next election. The results are below.

Respondent	Do you expect party A to win or lose against party B?
1	Lose
2	Lose
3	Lose
4	I don't know
5	Win
6	Lose
7	Win
8	I don't know
9	Lose
10	Lose

Make a cross tabulation from these survey results.

ANSWER

Below is the cross tabulation.

Response	Frequency	%
Win	2	20
I don't know	2	20
Lose	6	60
Sum	10	100

SUMMARY

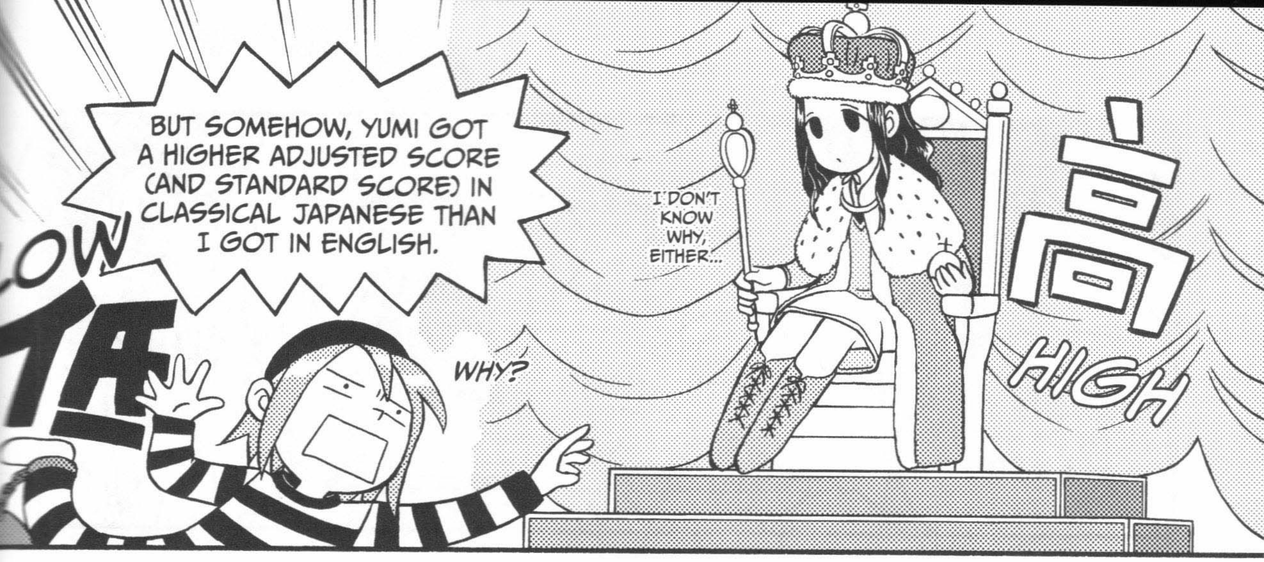
- One way to see the big picture of all the data is to make a cross tabulation.

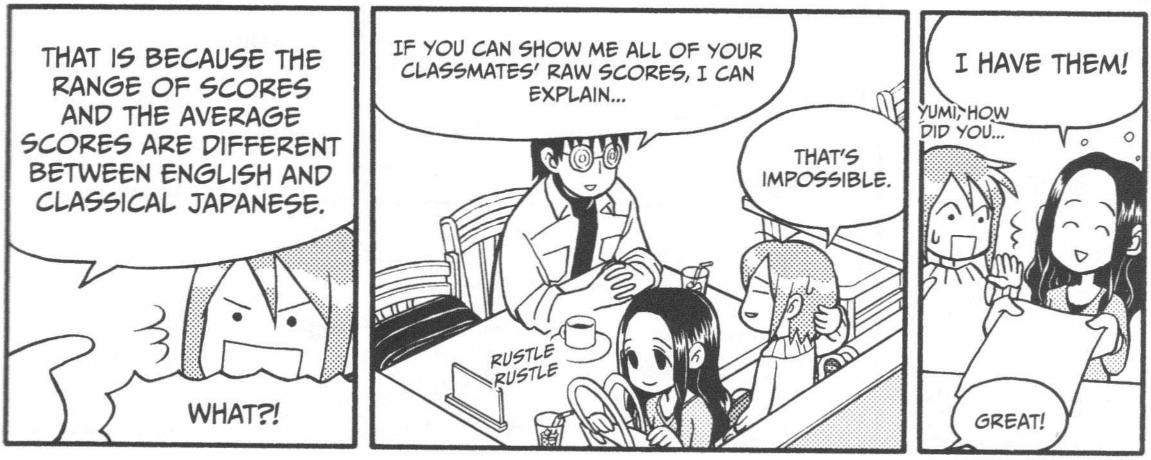
4

STANDARD SCORE AND DEVIATION SCORE

1. NORMALIZATION AND STANDARD SCORE

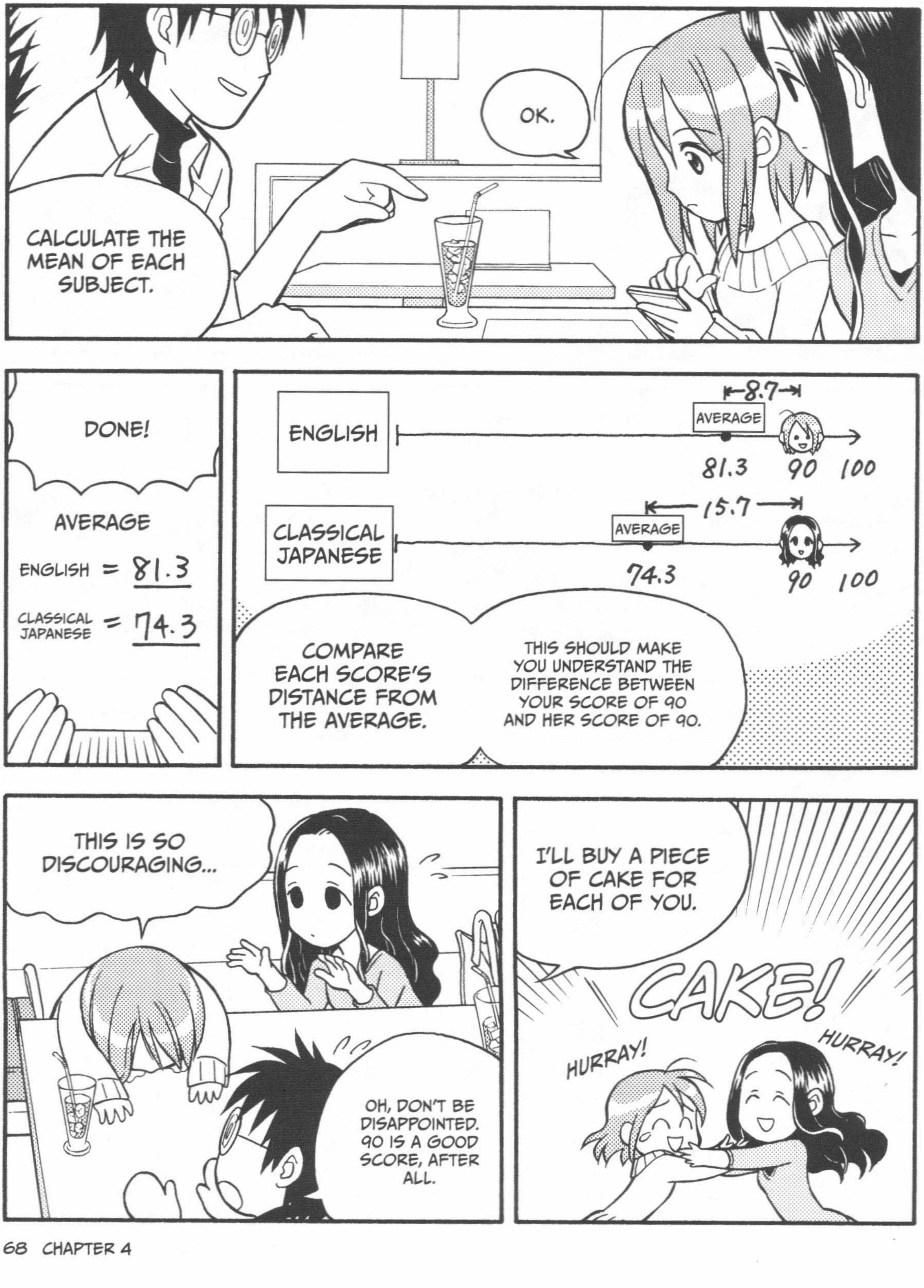
* ADJUSTING TEST RESULTS BASED ON STANDARD SCORE IS COMMONLY KNOWN AS *GRADING ON A CURVE.*



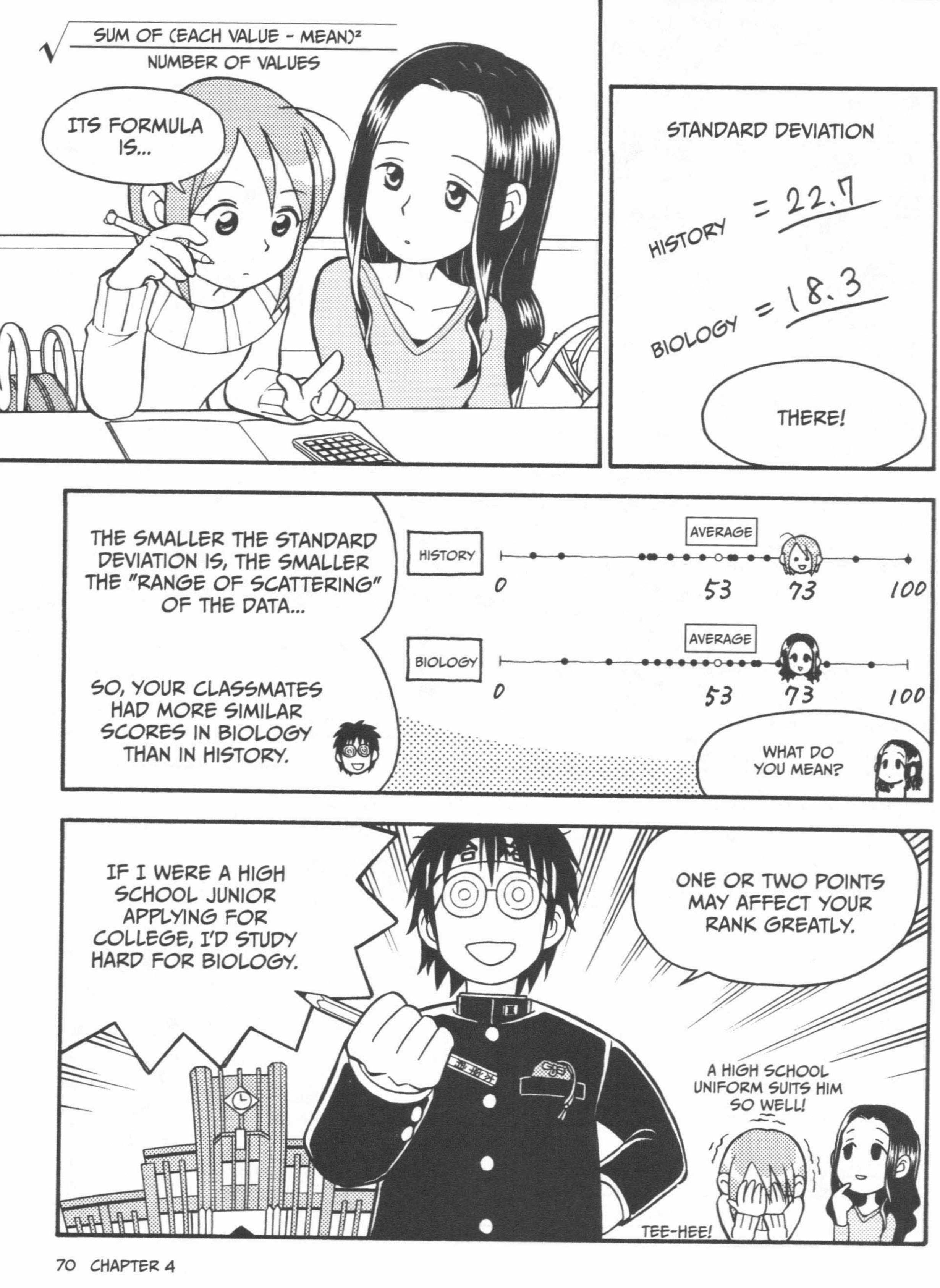


RAW TEST SCORES (OUT OF 100)

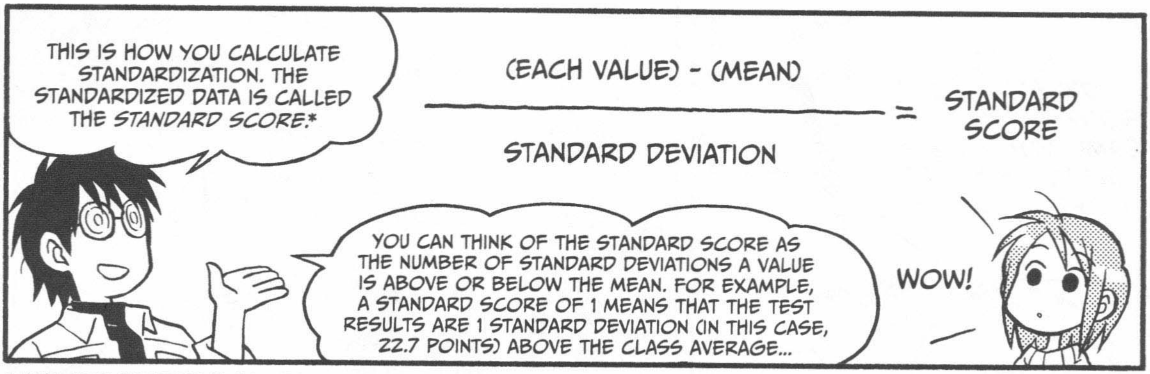
STUDENT	ENGLISH	CLASSICAL JAPANESE	STUDENT	ENGLISH	CLASSICAL JAPANESE
RUI	90	71	H	67	85
YUMI	81	90	I	87	93
A	73	79	J	78	89
B	97	70	K	85	78
C	85	67	L	96	74
D	60	66	M	77	65
E	74	60	N	100	78
F	64	83	O	92	53
G	72	57	P	86	80


CALCULATE THE MEAN OF EACH SUBJECT.
OK.
DONE!
AVERAGE
ENGLISH = 81.3
CLASSICAL JAPANESE = 74.3
ENGLISH
AVERAGE
8.7
81.3
90
100
CLASSICAL JAPANESE
AVERAGE
15.7
74.3
90
100
COMPARE EACH SCORE'S DISTANCE FROM THE AVERAGE.
THIS SHOULD MAKE YOU UNDERSTAND THE DIFFERENCE BETWEEN YOUR SCORE OF 90 AND HER SCORE OF 90.
THIS IS SO DISCOURAGING...
OH, DON'T BE DISAPPOINTED. 90 IS A GOOD SCORE, AFTER ALL.
I'LL BUY A PIECE OF CAKE FOR EACH OF YOU.
CAKE!
HURRAY!
HURRAY!

STUDENT	HISTORY	BIOLOGY	STUDENT	HISTORY	BIOLOGY
RUI	73	59	H	7	50
YUMI	61	73	I	53	41
A	14	47	J	100	62
B	41	38	K	57	44
C	49	63	L	45	26
D	87	56	M	56	91
E	69	15	N	34	35
F	65	53	O	37	53
G	36	80	P	70	68
			AVERAGE	53	53

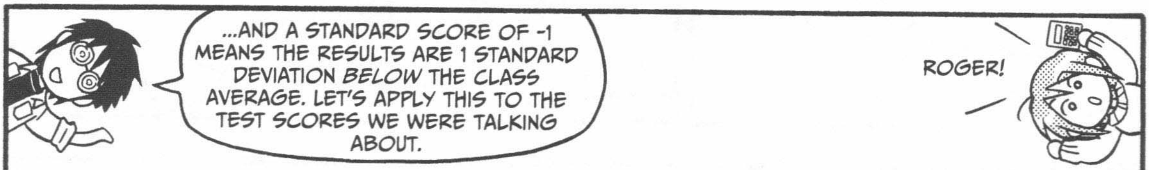

$\sqrt{\frac{\text{SUM OF (EACH VALUE - MEAN)}^2}{\text{NUMBER OF VALUES}}}$
ITS FORMULA IS...
STANDARD DEVIATION
HISTORY = 22.7
BIOLOGY = 18.3
THERE!
THE SMALLER THE STANDARD DEVIATION IS, THE SMALLER THE "RANGE OF SCATTERING" OF THE DATA...
HISTORY
AVERAGE
0
53
73
100
BIOLOGY
AVERAGE
0
53
73
100
SO, YOUR CLASSMATES HAD MORE SIMILAR SCORES IN BIOLOGY THAN IN HISTORY.
WHAT DO YOU MEAN?
IF I WERE A HIGH SCHOOL JUNIOR APPLYING FOR COLLEGE, I'D STUDY HARD FOR BIOLOGY.
ONE OR TWO POINTS MAY AFFECT YOUR RANK GREATLY.
A HIGH SCHOOL UNIFORM SUITS HIM SO WELL!
TEE-HEE!


NOW I UNDERSTAND...OUR SCORES WERE THE SAME, BUT YOUR 73 IN BIOLOGY WAS MORE VALUABLE...
CHEER UP, RUI!
SLUMP!
I DIDN'T KNOW THAT IT WOULD BE SO ANNOYING TO COMPARE SCORES.
(BACK UP AGAIN!)
I AGREE.
キラーン
SHE HAS A GOOD POINT!
THAT IS WHY *NORMALIZATION* WAS INTRODUCED.
IT IS ALSO CALLED *STANDARDIZATION.*
NORMALIZATION!?
IT IS A CALCULATION USING THE DISTANCE FROM THE MEAN AND THE STANDARD DEVIATION OF THE DATA...
THAT MAKES IT EASIER TO FIGURE OUT HOW MUCH A SCORE IS WORTH.



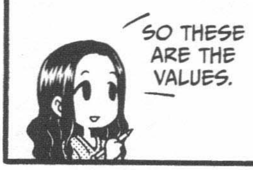
$$\frac{\text{(EACH VALUE)} - \text{(MEAN)}}{\text{STANDARD DEVIATION}} = \text{STANDARD SCORE}$$

* STANDARD SCORE IS ALSO CALLED Z-SCORE.



RESULTS AND STANDARD SCORES OF HISTORY AND BIOLOGY TESTS

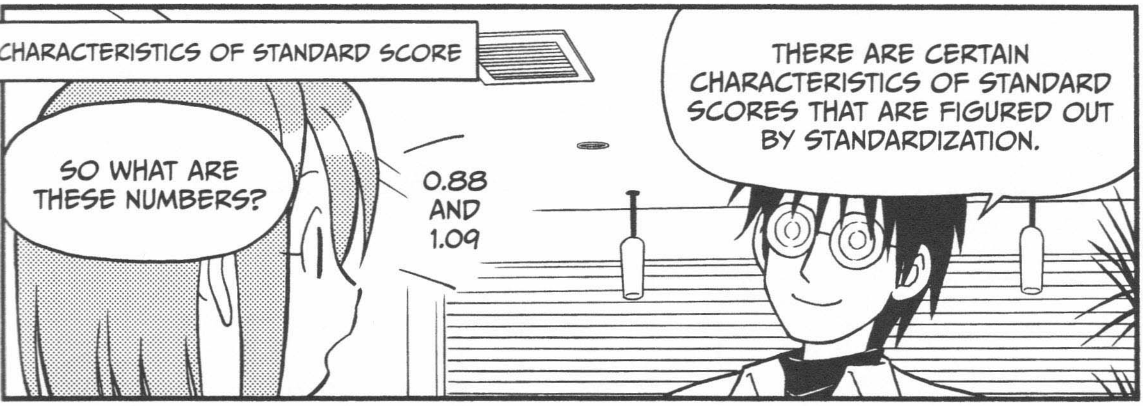
STUDENT	HISTORY	BIOLOGY	STANDARD SCORE OF HISTORY	STANDARD SCORE OF BIOLOGY
RUI	73	59	0.88	0.33
YUMI	61	73	0.35	1.09
A	14	47	-1.71	-0.33
B	41	38	-0.53	-0.82
C	49	63	-0.18	0.55
D	87	56	1.49	0.16
E	69	15	0.70	-2.08
F	65	53	0.53	0
G	36	80	-0.75	1.48
H	7	50	-2.02	-0.16
I	53	41	0	-0.66
J	100	62	2.07	0.49
K	57	44	0.18	-0.49
L	45	26	-0.35	-1.48
M	56	91	0.13	2.08
N	34	35	-0.84	-0.98
O	37	53	-0.70	0
P	70	68	0.75	0.82
AVERAGE	53	53	0	0
STANDARD DEVIATION	22.7	18.3	1	1



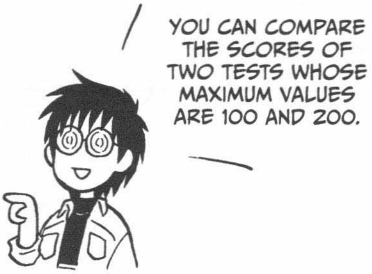
STANDARD SCORE OF RUI'S HISTORY TEST $\frac{73-53}{22.7} = \frac{20}{22.7} = 0.88$

STANDARD SCORE OF YUMI'S BIOLOGY TEST $\frac{73-53}{18.3} = \frac{20}{18.3} = 1.09$

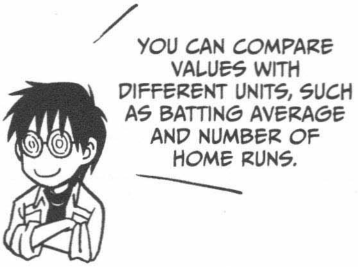
2. CHARACTERISTICS OF STANDARD SCORE

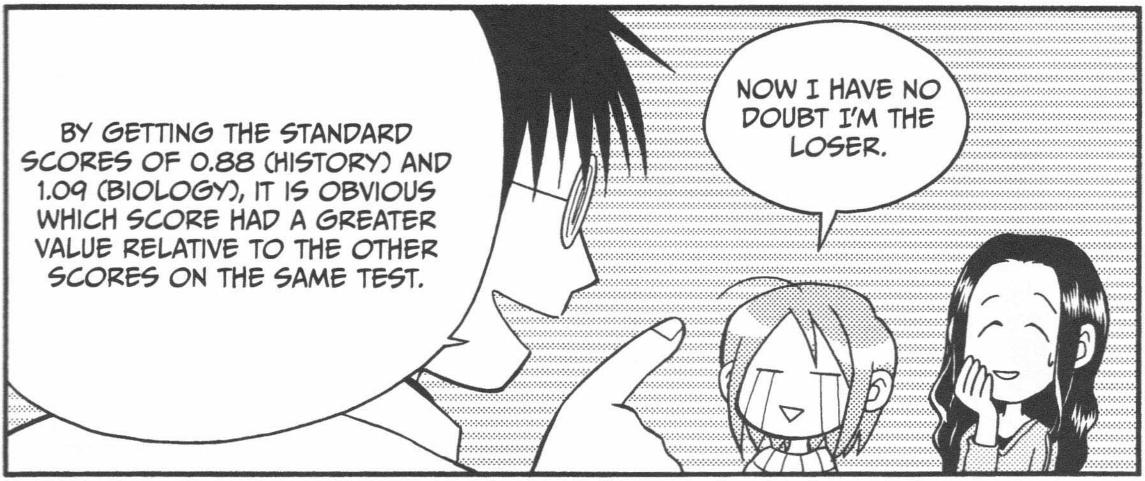


(1) No matter what the maximum value of your variable is, the arithmetic mean of the standard score is always 0, and the standard deviation is always 1.

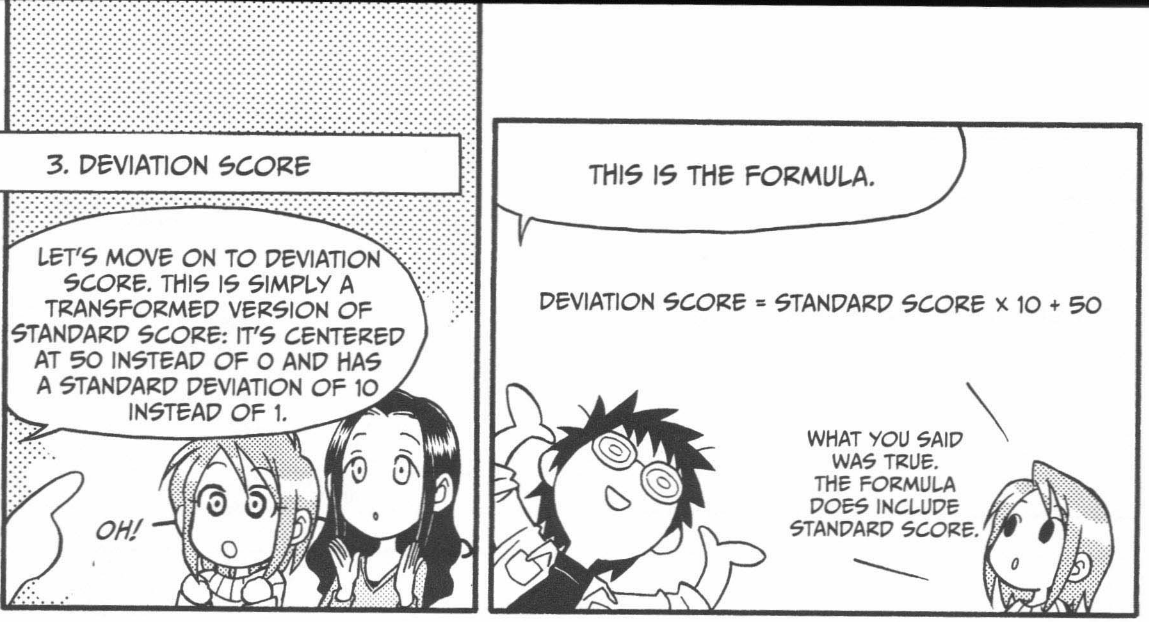


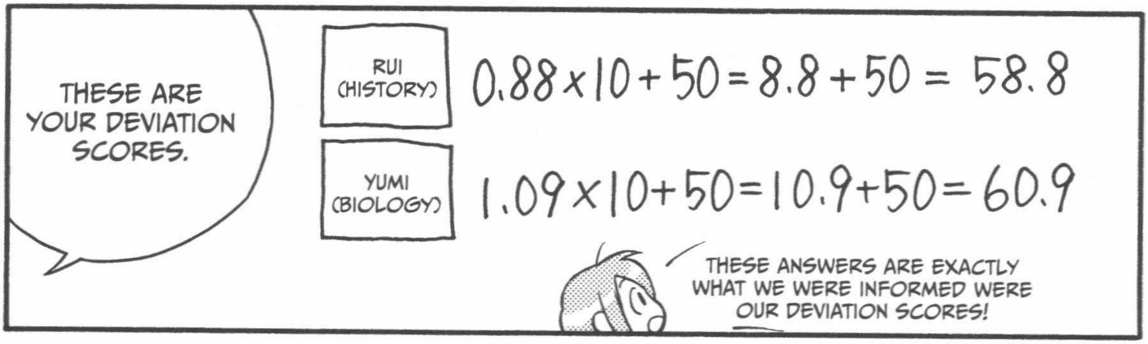
(2) Whatever the unit of the variable in question is, the arithmetic mean of the standard score is always 0, and the standard deviation is always 1.



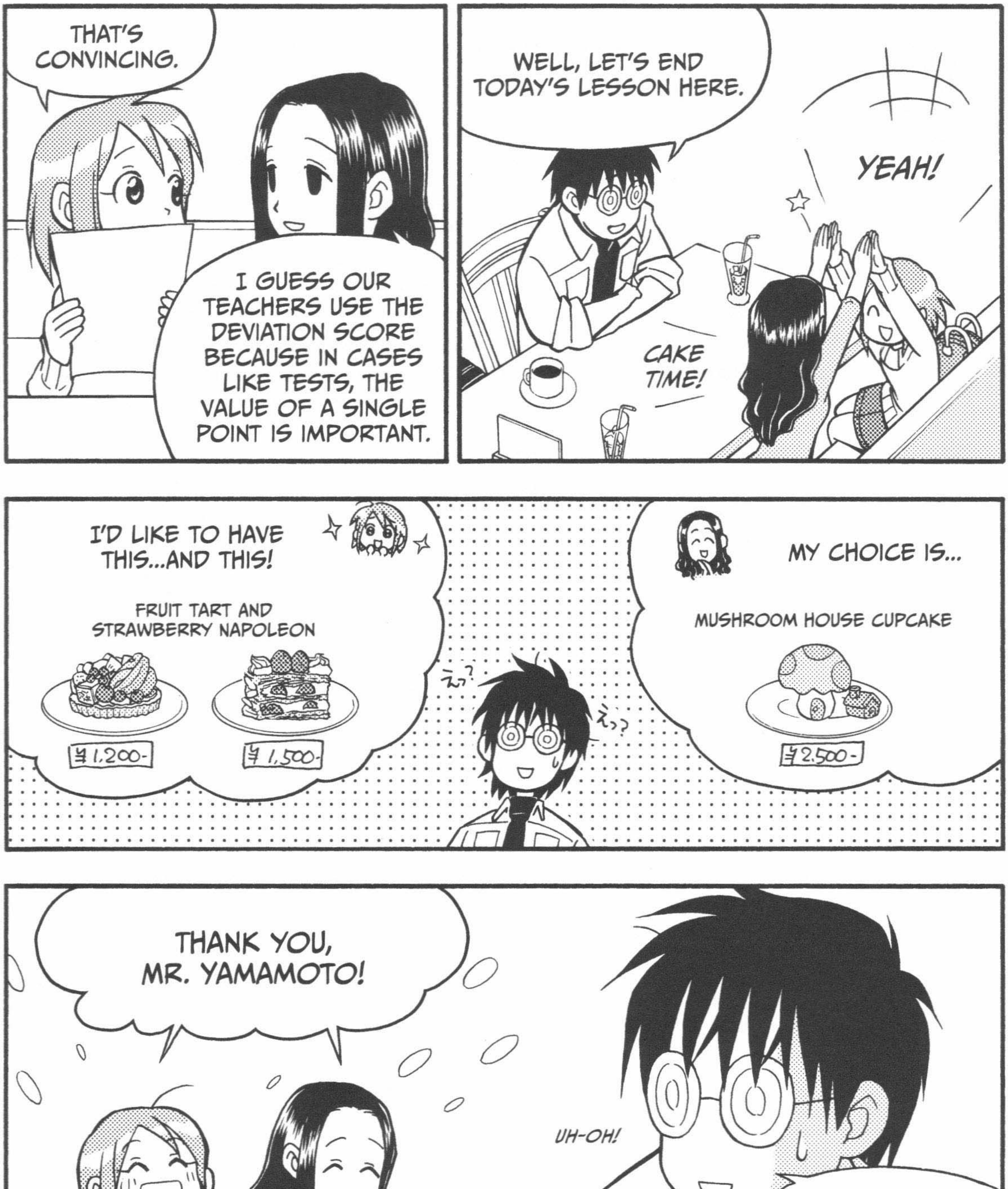


3. DEVIATION SCORE





STANDARD SCORE	(1) No matter what the maximum value of your variable is, the arithmetic mean of the standard score is always 0, and the standard deviation is always 1. (2) Whatever the unit of the variable in question is, the arithmetic mean of the standard score is always 0, and the standard deviation is always 1.
DEVIATION SCORE	(1) No matter what the maximum value of your variable is, the arithmetic mean of the deviation score is always 50, and its standard deviation is always 10. (2) No matter what units of measurement your variable uses, the arithmetic mean of the deviation score is always 50, and its standard deviation is always 10.


THAT'S CONVINCING.
I GUESS OUR TEACHERS USE THE DEVIATION SCORE BECAUSE IN CASES LIKE TESTS, THE VALUE OF A SINGLE POINT IS IMPORTANT.
WELL, LET'S END TODAY'S LESSON HERE.
YEAH!
CAKE TIME!
I'D LIKE TO HAVE THIS...AND THIS!
FRUIT TART AND STRAWBERRY NAPOLEON
¥1,200-
¥1,500-
えっ?
えっ?
MY CHOICE IS...
MUSHROOM HOUSE CUPCAKE
¥2,500-
THANK YOU, MR. YAMAMOTO!
UH-OH!
I HOPE I CAN AFFORD THIS.

4. INTERPRETATION OF DEVIATION SCORE

Special caution is necessary when interpreting deviation scores. As explained on page 74, the definition of deviation score is:

$$\text{deviation score} = \text{standard score} \times 10 + 50 = \frac{(\text{each value} - \text{mean})}{\text{standard deviation}} \times 10 + 50$$

As mentioned on page 62, Rui's class has a total of 40 students, and as mentioned on page 40, there are 18 girls in the class. The example of deviation score on page 69 is not for the whole class, but is for the girls only. If the story were about the whole class, the mean and standard deviation would have been different from those for the girls only. Naturally, the deviation scores for Rui and Yumi would have been different as well. In fact, when everybody in the class is taken into consideration, Rui has the higher deviation score. Table 4-1 shows the test results for the whole class. Try calculating the deviation score.

To tell you the answer in advance, the deviation score for Rui's history test is 59.1, and that of Yumi's biology test is 56.7.

Suppose the same test is given to students in classes 1 and 2. The mean and standard deviation of class 1 are calculated individually, and deviation scores are obtained according to those amounts. Similarly, mean, standard deviation, and deviation scores for class 2 are obtained. Student A in class 1 has a deviation score of 57. Student B in class 2 has the same deviation score of 57. Outwardly, students A and B seem to have the same ability. However, the mean and standard deviation used to calculate these two deviation scores differ, because they come from two different classes. Unless the mean and standard deviation of the two classes are equal, you cannot compare the deviation scores of the two students.

Here is another example. Suppose student A takes an entrance exam at a prep school in April and gets a deviation score of 54. After studying hard at a special summer course, student A takes an entrance exam at a different prep school in September. The deviation score is 62. It may seem that student A's proficiency has increased. However, the exam and the students taking it in April are different from the exam and the students taking it in September. Therefore, you cannot compare the deviation scores for these two exams, because the data used to calculate the mean and standard deviation of the April and September exams is different. In exam situations, you can only compare deviation scores for a group of students who all take the same exam. Keep these facts in mind when you interpret deviation scores.

TABLE 4-1: TEST RESULTS OF HISTORY AND BIOLOGY (ALL MEMBERS OF RUI'S CLASS)

Girls	**History**	**Biology**	**Boys**	**History**	**Biology**
Rui	73	59	a	54	2
Yumi	61	73	b	93	7
A	14	47	c	91	98
B	41	38	d	37	85
C	49	63	e	44	100
D	87	56	f	16	29
E	69	15	g	12	57
F	65	53	h	44	37
G	36	80	i	4	95
H	7	50	j	17	39
I	53	41	k	66	70
J	100	62	l	53	14
K	57	44	m	14	97
L	45	26	n	73	39
M	56	91	o	6	75
N	34	35	p	22	80
O	37	53	q	69	77
P	70	68	r	95	14
			s	16	24
			t	37	91
			u	14	36
			v	88	76

Average of the whole class	48.0	54.9
Standard deviation of the whole class	27.5	26.9

EXERCISE AND ANSWER

EXERCISE

Below are the results of a high school girls' 100m track race.

Runner	100m track race (seconds)
Ms. A	16.3
Ms. B	22.4
Ms. C	18.5
Ms. D	18.7
Ms. E	20.1
Mean	19.2
Standard deviation	2.01

1. Demonstrate that the mean of the standard scores of the 100m track race is 0.
2. Demonstrate that the standard deviation of the standard score of the 100m track race is 1.

ANSWER

1. Mean of the standard score of the 100m track race

$$= \frac{\left(\frac{16.3 - 19.2}{2.01}\right) + \left(\frac{22.4 - 19.2}{2.01}\right) + \left(\frac{18.5 - 19.2}{2.01}\right) + \left(\frac{18.7 - 19.2}{2.01}\right) + \left(\frac{20.1 - 19.2}{2.01}\right)}{5}$$

$$= \frac{\left\{\frac{(16.3 - 19.2) + (22.4 - 19.2) + (18.5 - 19.2) + (18.7 - 19.2) + (20.1 - 19.2)}{2.01}\right\}}{5}$$

The numerator has been clarified.

$$= \frac{\left\{\frac{16.3 + 22.4 + 18.5 + 18.7 + 20.1 - 19.2 - 19.2 - 19.2 - 19.2 - 19.2}{2.01}\right\}}{5}$$

The numerator has been reorganized so that each value and (-19.2) are separate.

$$= \frac{\left\{\frac{96 - 19.2 \times 5}{2.01}\right\}}{5}$$

$$= \frac{\left\{\frac{96 - 96}{2.01}\right\}}{5}$$

$$= \frac{0}{5}$$

$$= 0$$

2. Standard deviation of the standard score of the 100m track race

$$= \sqrt{\frac{\left(\frac{16.3 - 19.2}{2.01} - 0\right)^2 + \left(\frac{22.4 - 19.2}{2.01} - 0\right)^2 + \left(\frac{18.5 - 19.2}{2.01} - 0\right)^2 + \left(\frac{18.7 - 19.2}{2.01} - 0\right)^2 + \left(\frac{20.1 - 19.2}{2.01} - 0\right)^2}{5}}$$

$$= \sqrt{\frac{\left(\frac{16.3 - 19.2}{2.01}\right)^2 + \left(\frac{22.4 - 19.2}{2.01}\right)^2 + \left(\frac{18.5 - 19.2}{2.01}\right)^2 + \left(\frac{18.7 - 19.2}{2.01}\right)^2 + \left(\frac{20.1 - 19.2}{2.01}\right)^2}{5}}$$

$$= \sqrt{\frac{\left\{\frac{(16.3 - 19.2)^2 + (22.4 - 19.2)^2 + (18.5 - 19.2)^2 + (18.7 - 19.2)^2 + (20.1 - 19.2)^2}{2.01^2}\right\}}{5}}$$

The numerator has been clarified.

$$= \sqrt{\frac{1}{2.01^2} \times \frac{(16.3 - 19.2)^2 + (22.4 - 19.2)^2 + (18.5 - 19.2)^2 + (18.7 - 19.2)^2 + (20.1 - 19.2)^2}{5}}$$

The numerator has been clarified.

$$= \frac{1}{2.01} \times \sqrt{\frac{(16.3 - 19.2)^2 + (22.4 - 19.2)^2 + (18.5 - 19.2)^2 + (18.7 - 19.2)^2 + (20.1 - 19.2)^2}{5}}$$

$$= \frac{1}{\text{standard deviation of the 100m track race}} \times \text{standard deviation of the 100m track race}$$

Carefully look at the table on page 78.

$$= 1$$

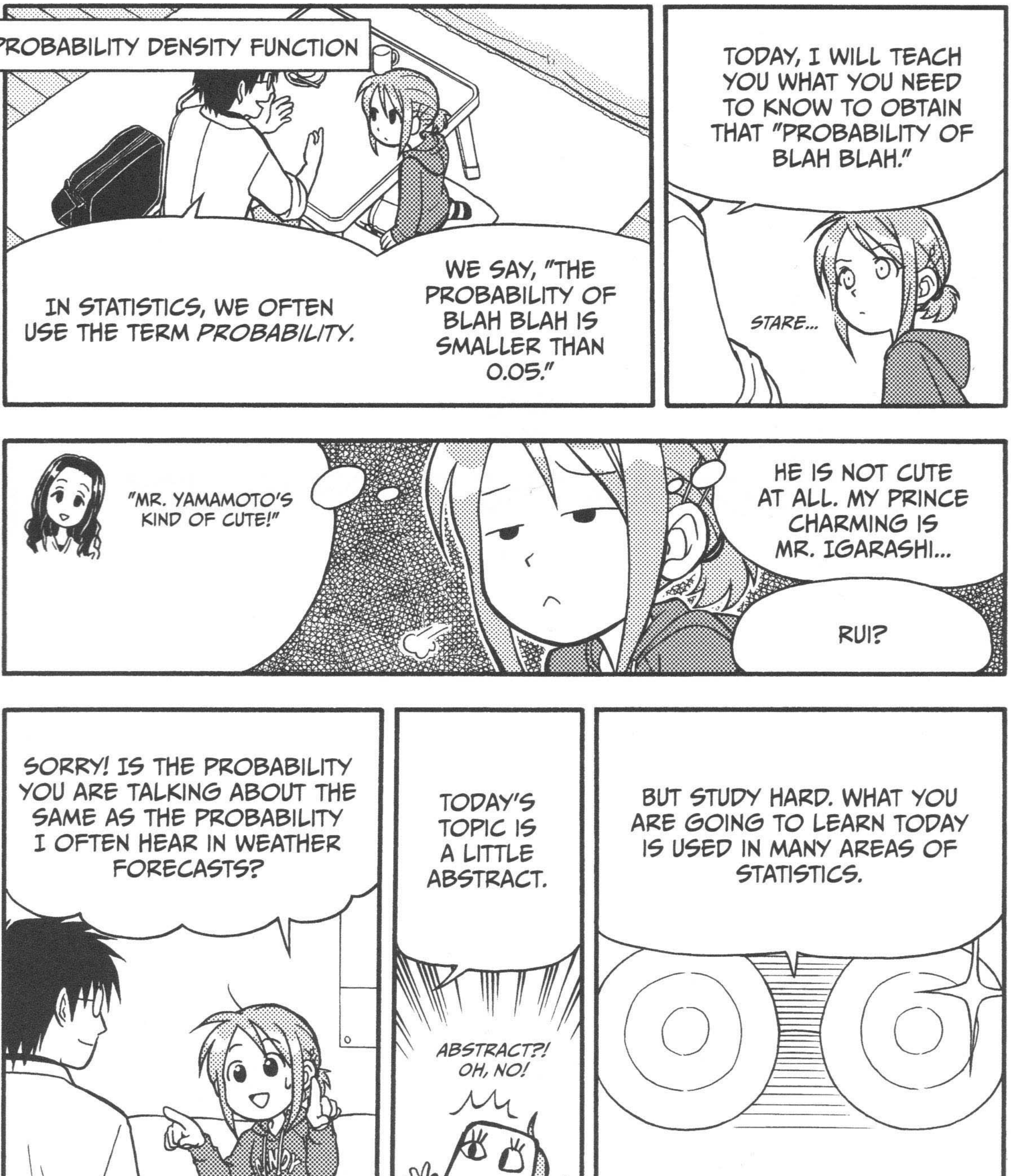
SUMMARY

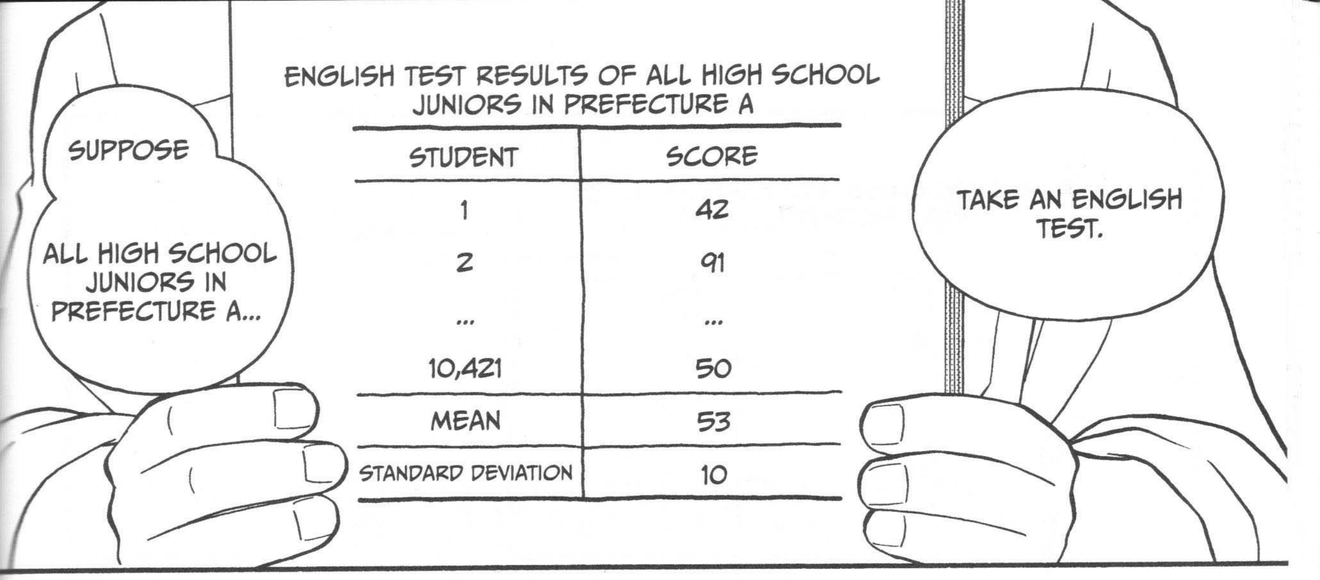
- *Standardization* helps you examine the value of a data point relative to the rest of your data by using its distance from the mean and "the size of scattering" of the data.
- Use standardization to:
  - Compare variables with different ranges
  - Compare variables that use different units of measurements
- A data point that has been standardized is called the *standard score* for that observation. Deviation score is an application of standard score.

5

LET'S OBTAIN THE PROBABILITY

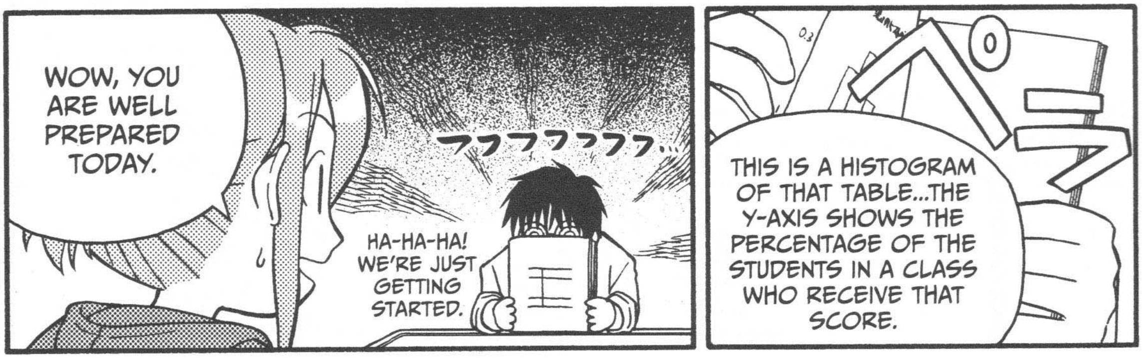
1. PROBABILITY DENSITY FUNCTION

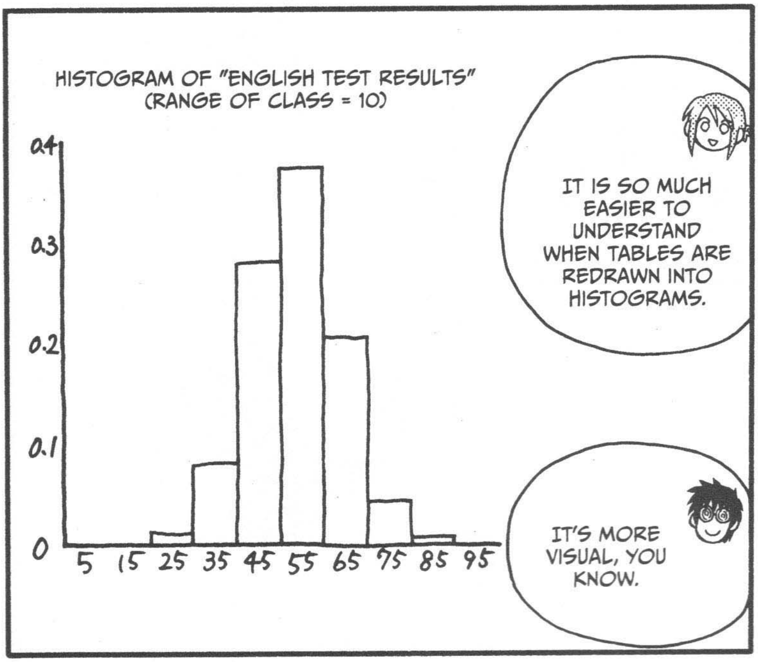


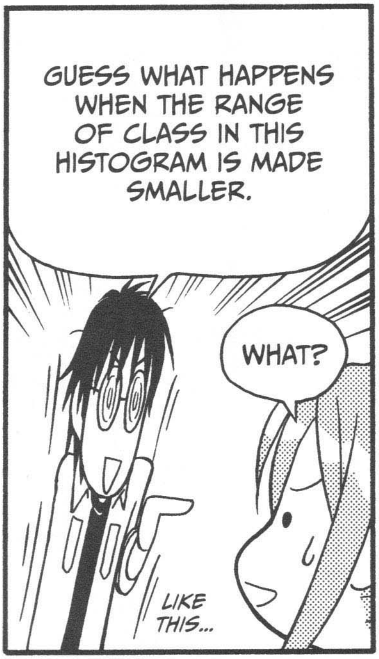


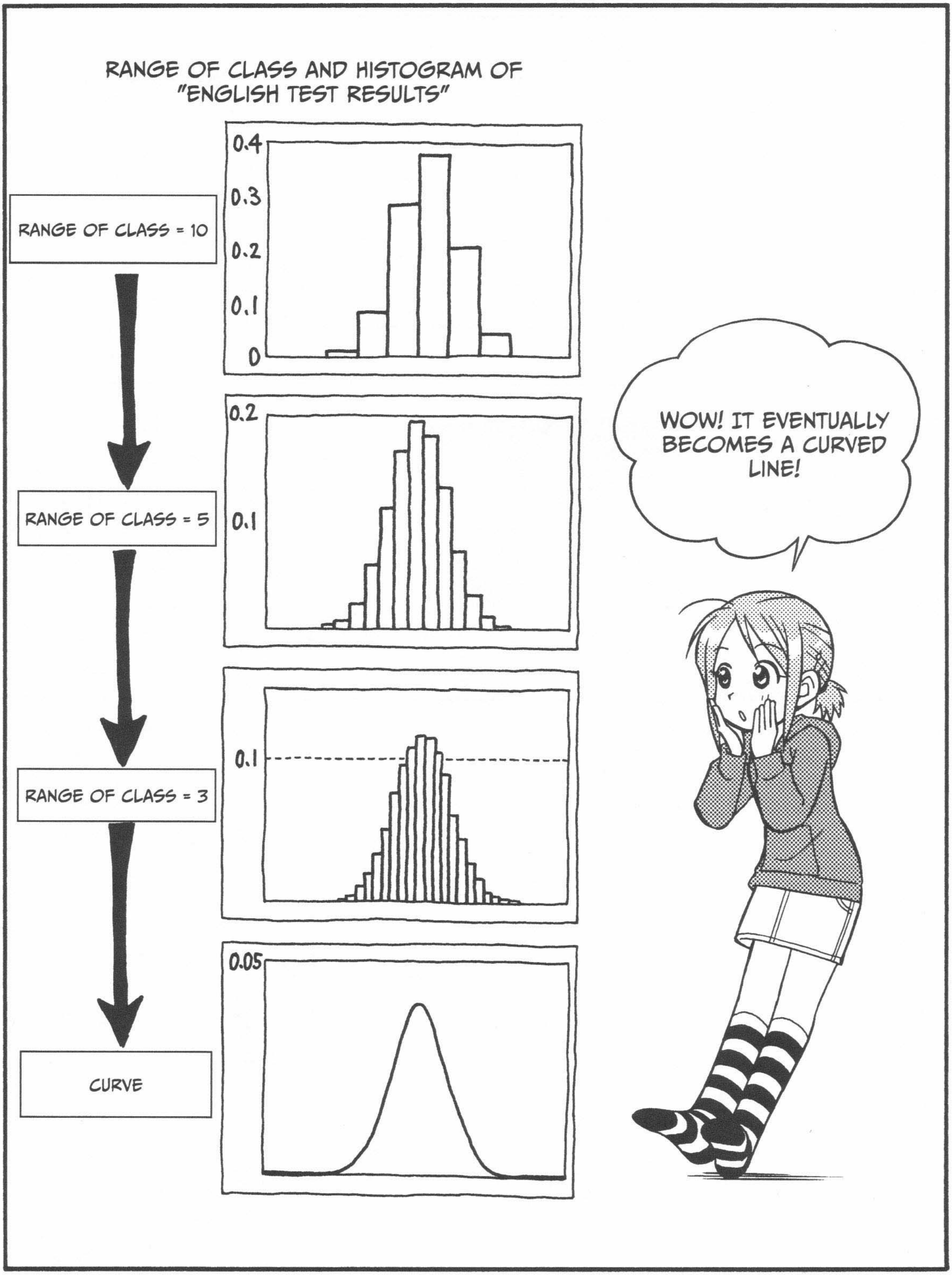
ENGLISH TEST RESULTS OF ALL HIGH SCHOOL JUNIORS IN PREFECTURE A

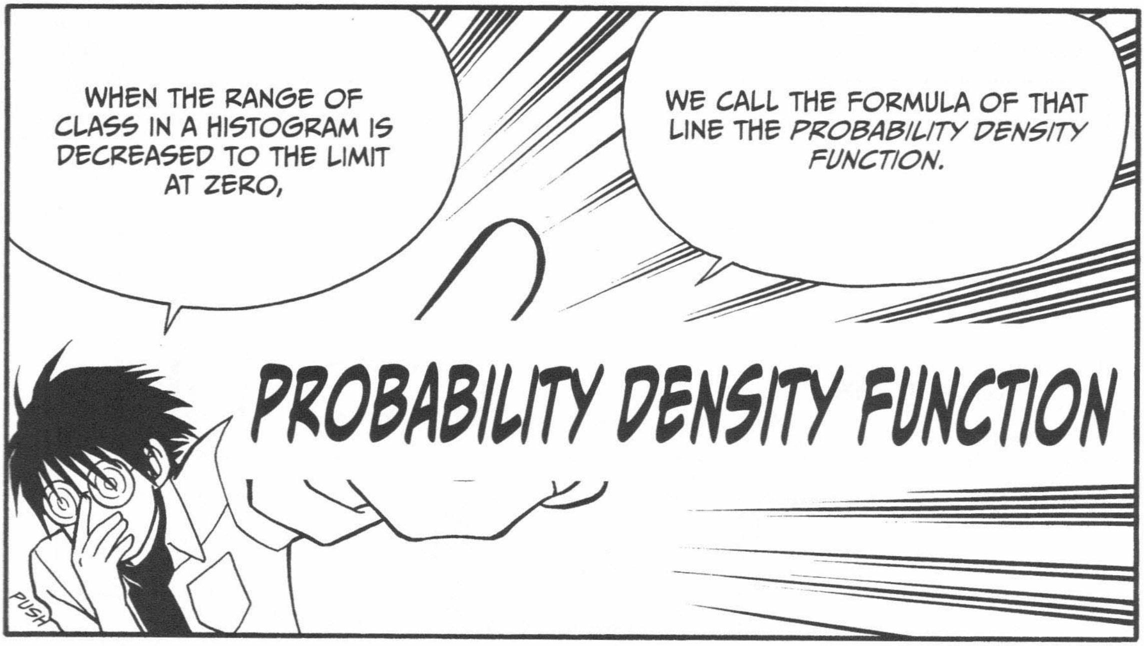
STUDENT	SCORE
1	42
2	91
...	...
10,421	50
MEAN	53
STANDARD DEVIATION	10

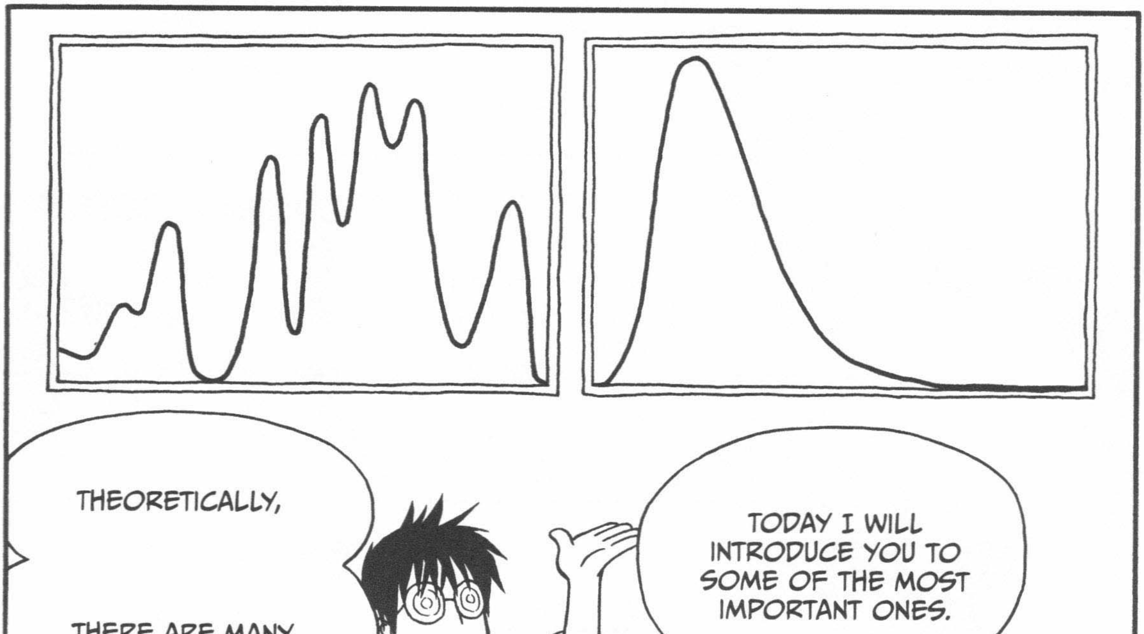






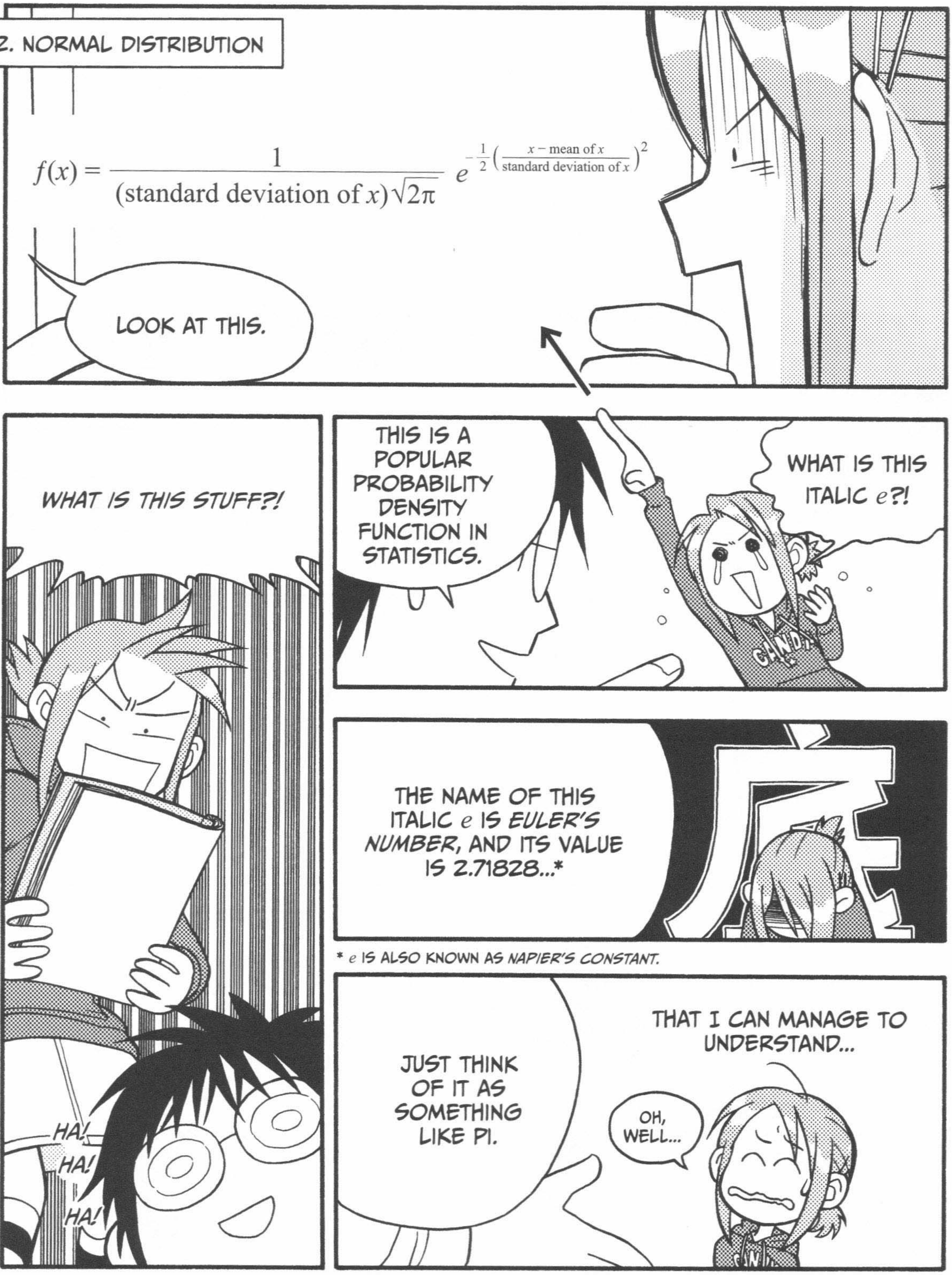

RANGE OF CLASS AND HISTOGRAM OF "ENGLISH TEST RESULTS"
RANGE OF CLASS = 10
0.4
0.3
0.2
0.1
0
RANGE OF CLASS = 5
0.2
0.1
RANGE OF CLASS = 3
0.1
CURVE
0.05
WOW! IT EVENTUALLY BECOMES A CURVED LINE!


WHEN THE RANGE OF CLASS IN A HISTOGRAM IS DECREASED TO THE LIMIT AT ZERO,
WE CALL THE FORMULA OF THAT LINE THE *PROBABILITY DENSITY FUNCTION.*
PROBABILITY DENSITY FUNCTION
PUSH


THEORETICALLY,
THERE ARE MANY TYPES OF PROBABILITY DENSITY FUNCTION GRAPHS.
TODAY I WILL INTRODUCE YOU TO SOME OF THE MOST IMPORTANT ONES.
PLEASE GO AHEAD.

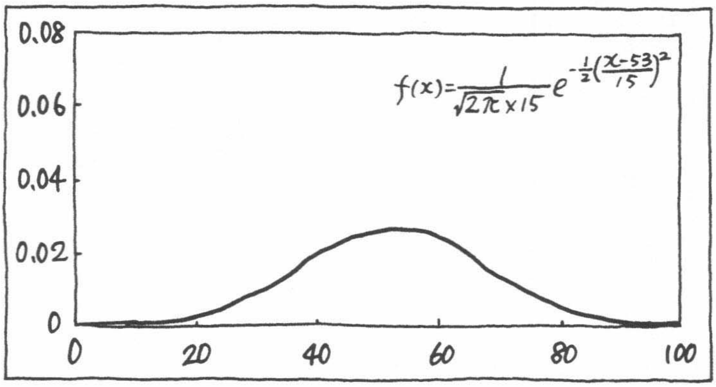
2. NORMAL DISTRIBUTION

$$f(x) = \frac{1}{(\text{standard deviation of } x)\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x - \text{mean of } x}{\text{standard deviation of } x}\right)^2}$$

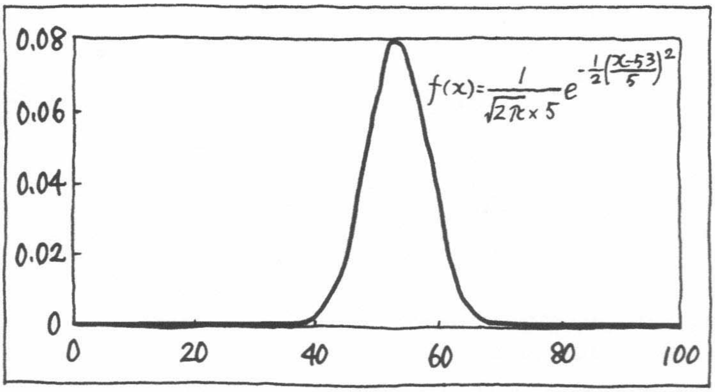


* *e* IS ALSO KNOWN AS *NAPIER'S CONSTANT.*

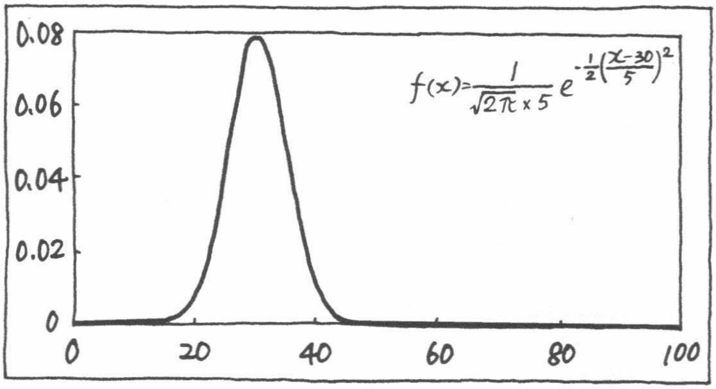
MEAN IS 53, STANDARD DEVIATION IS 15

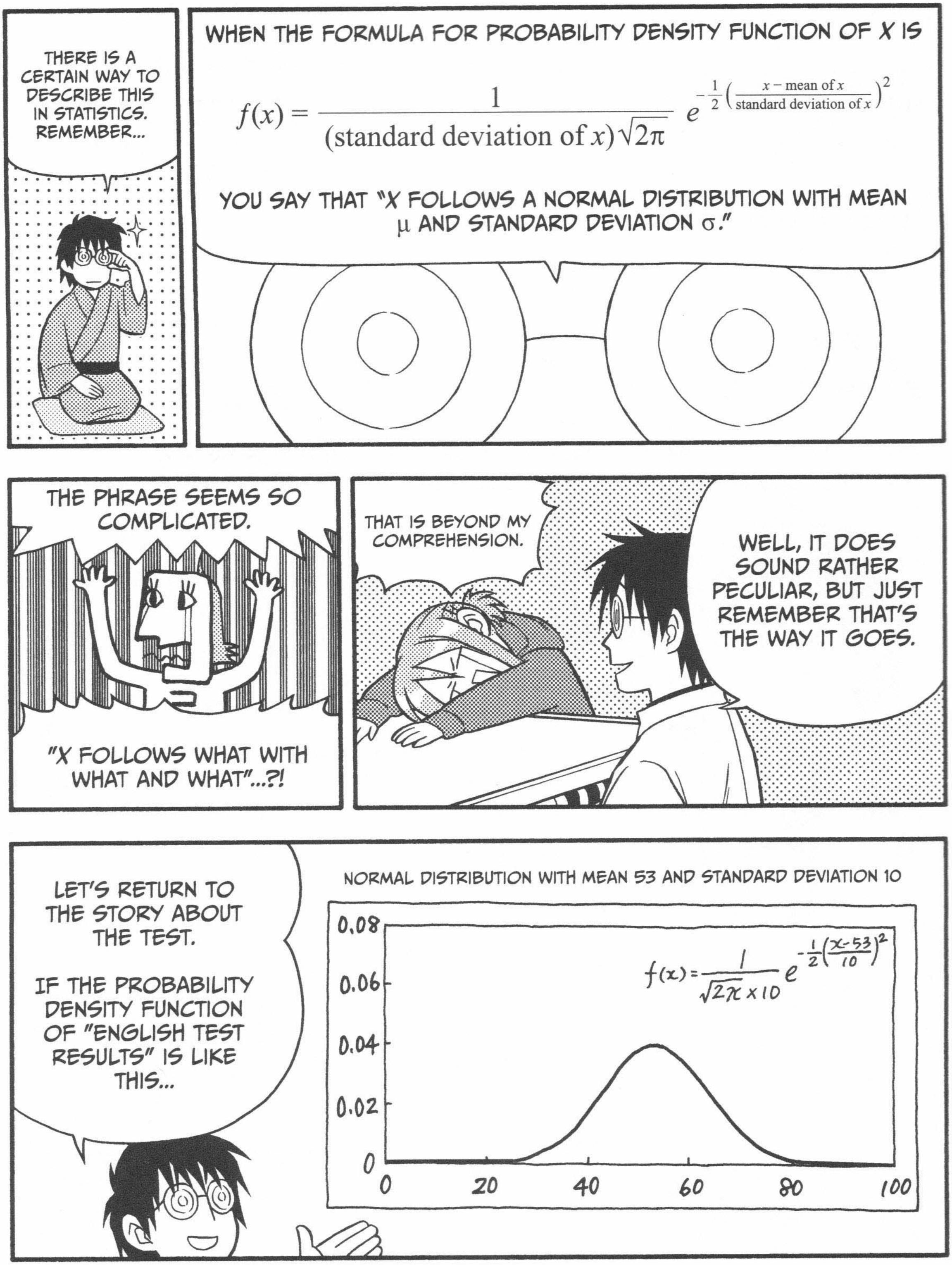


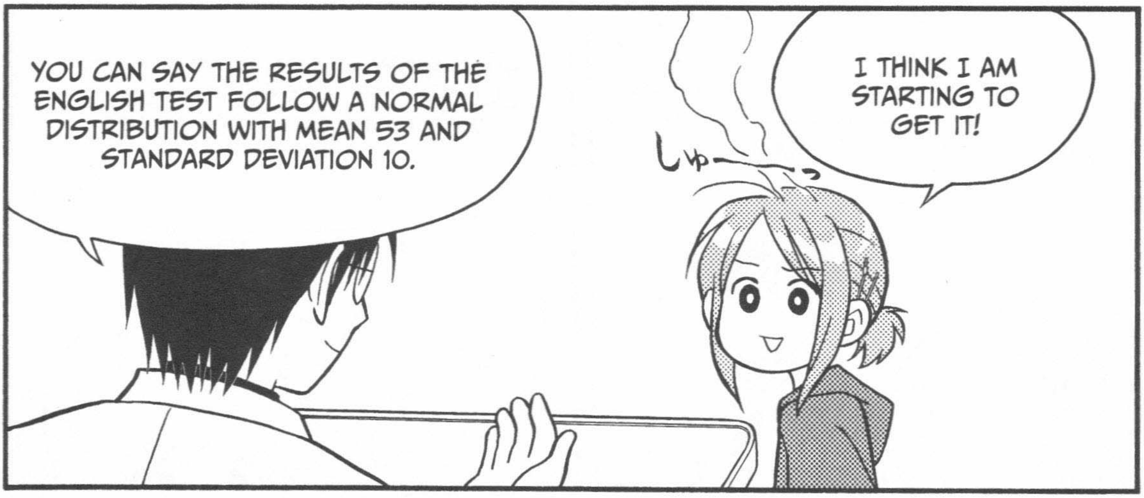
MEAN IS 53, STANDARD DEVIATION IS 5



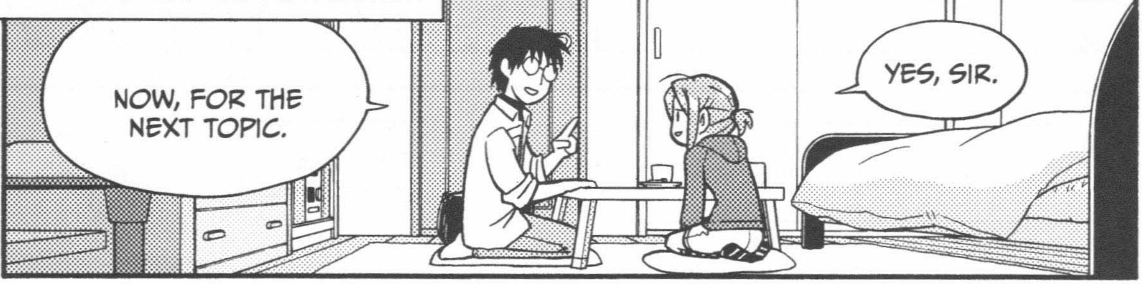
MEAN IS 30, STANDARD DEVIATION IS 5

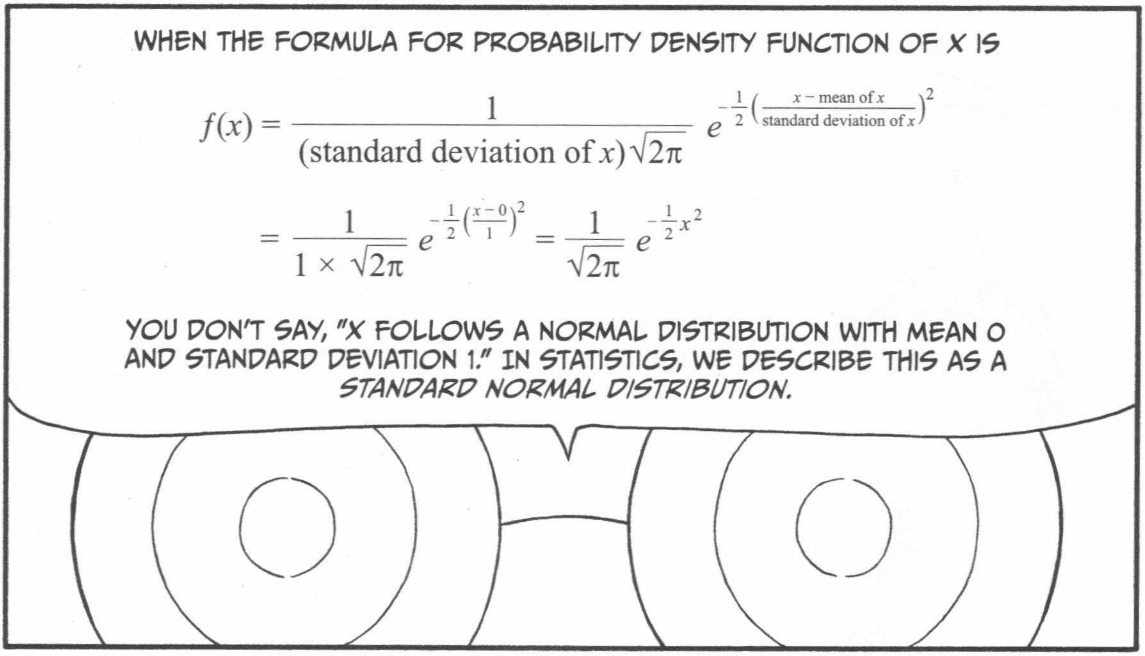



THERE IS A CERTAIN WAY TO DESCRIBE THIS IN STATISTICS. REMEMBER...
WHEN THE FORMULA FOR PROBABILITY DENSITY FUNCTION OF X IS
$f(x) = \frac{1}{(\text{standard deviation of } x)\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x - \text{mean of } x}{\text{standard deviation of } x}\right)^2}$
YOU SAY THAT "X FOLLOWS A NORMAL DISTRIBUTION WITH MEAN μ AND STANDARD DEVIATION σ."
THE PHRASE SEEMS SO COMPLICATED.
"X FOLLOWS WHAT WITH WHAT AND WHAT"...?!
THAT IS BEYOND MY COMPREHENSION.
WELL, IT DOES SOUND RATHER PECULIAR, BUT JUST REMEMBER THAT'S THE WAY IT GOES.
LET'S RETURN TO THE STORY ABOUT THE TEST.
IF THE PROBABILITY DENSITY FUNCTION OF "ENGLISH TEST RESULTS" IS LIKE THIS...
NORMAL DISTRIBUTION WITH MEAN 53 AND STANDARD DEVIATION 10
$f(x) = \frac{1}{\sqrt{2\pi} \times 10} e^{-\frac{1}{2}\left(\frac{x-53}{10}\right)^2}$
0.08
0.06
0.04
0.02
0
0
20
40
60
80
100



3. STANDARD NORMAL DISTRIBUTION



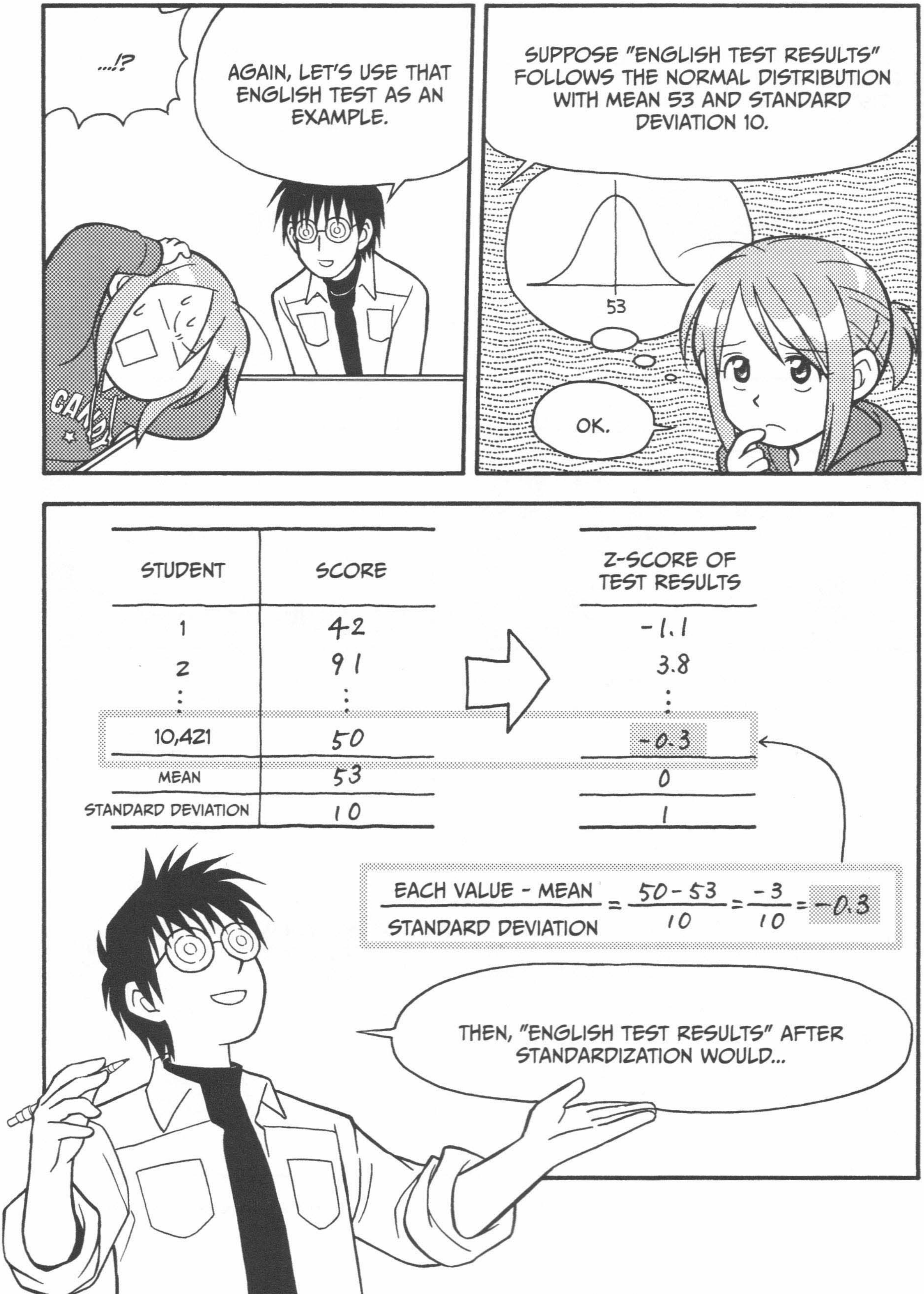


WHEN THE FORMULA FOR PROBABILITY DENSITY FUNCTION OF X IS

$$f(x) = \frac{1}{(\text{standard deviation of } x)\sqrt{2\pi}} \, e^{-\frac{1}{2}\left(\frac{x - \text{mean of } x}{\text{standard deviation of } x}\right)^2}$$

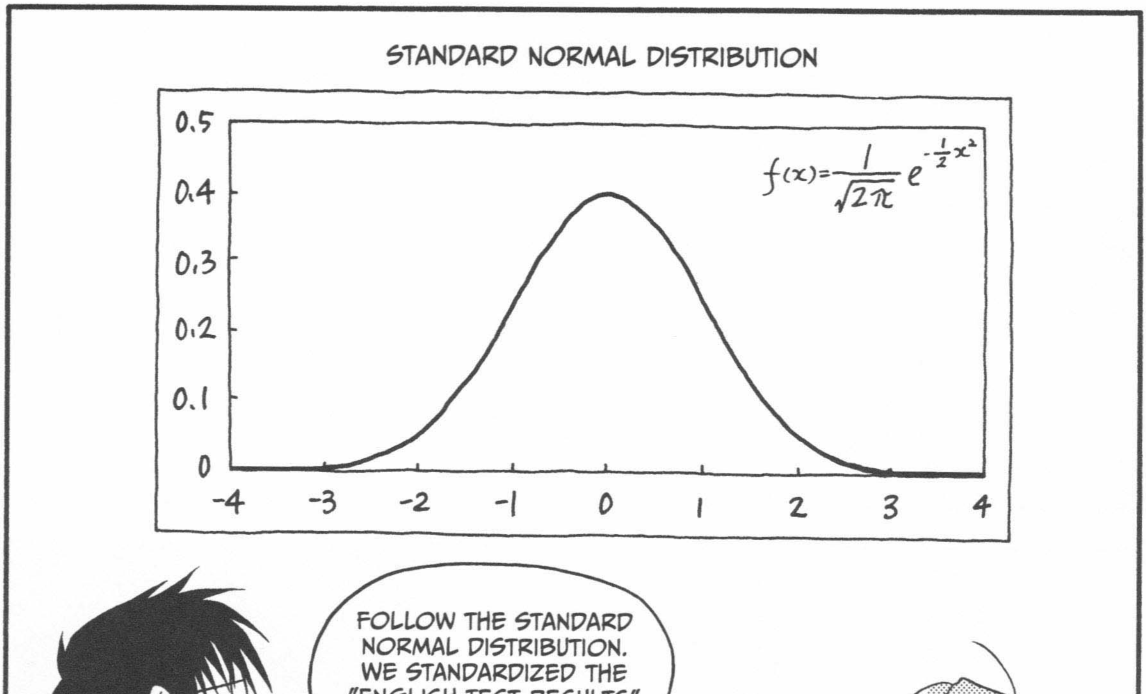
$$= \frac{1}{1 \times \sqrt{2\pi}} \, e^{-\frac{1}{2}\left(\frac{x-0}{1}\right)^2} = \frac{1}{\sqrt{2\pi}} \, e^{-\frac{1}{2}x^2}$$

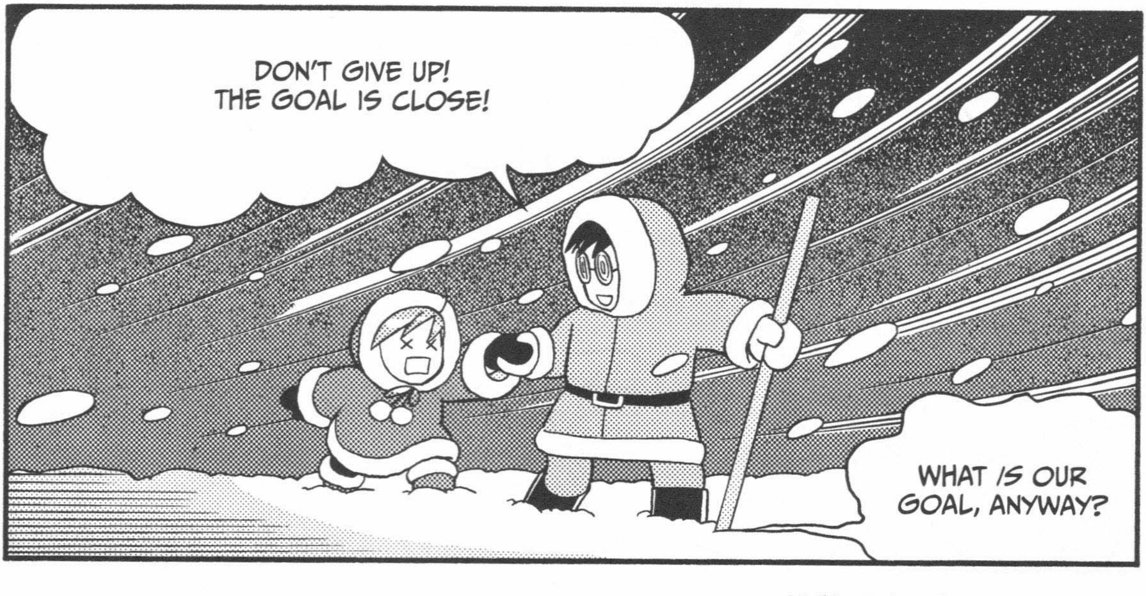
YOU DON'T SAY, "X FOLLOWS A NORMAL DISTRIBUTION WITH MEAN 0 AND STANDARD DEVIATION 1." IN STATISTICS, WE DESCRIBE THIS AS A *STANDARD NORMAL DISTRIBUTION.*



STUDENT	SCORE	Z-SCORE OF TEST RESULTS
1	42	-1.1
2	91	3.8
⋮	⋮	⋮
10,421	50	-0.3
MEAN	53	0
STANDARD DEVIATION	10	1

$$\frac{\text{EACH VALUE - MEAN}}{\text{STANDARD DEVIATION}} = \frac{50-53}{10} = \frac{-3}{10} = -0.3$$


STANDARD NORMAL DISTRIBUTION
$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4
FOLLOW THE STANDARD NORMAL DISTRIBUTION. WE STANDARDIZED THE "ENGLISH TEST RESULTS" DATA.
WOW, THAT LOOKS FAMILIAR!


DON'T GIVE UP! THE GOAL IS CLOSE!
WHAT *IS* OUR GOAL, ANYWAY?

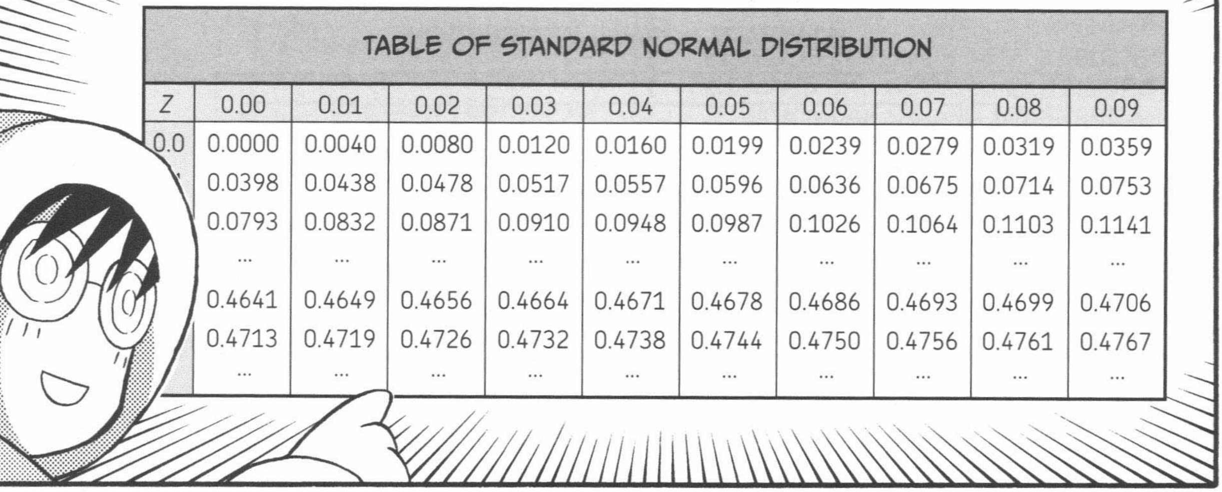
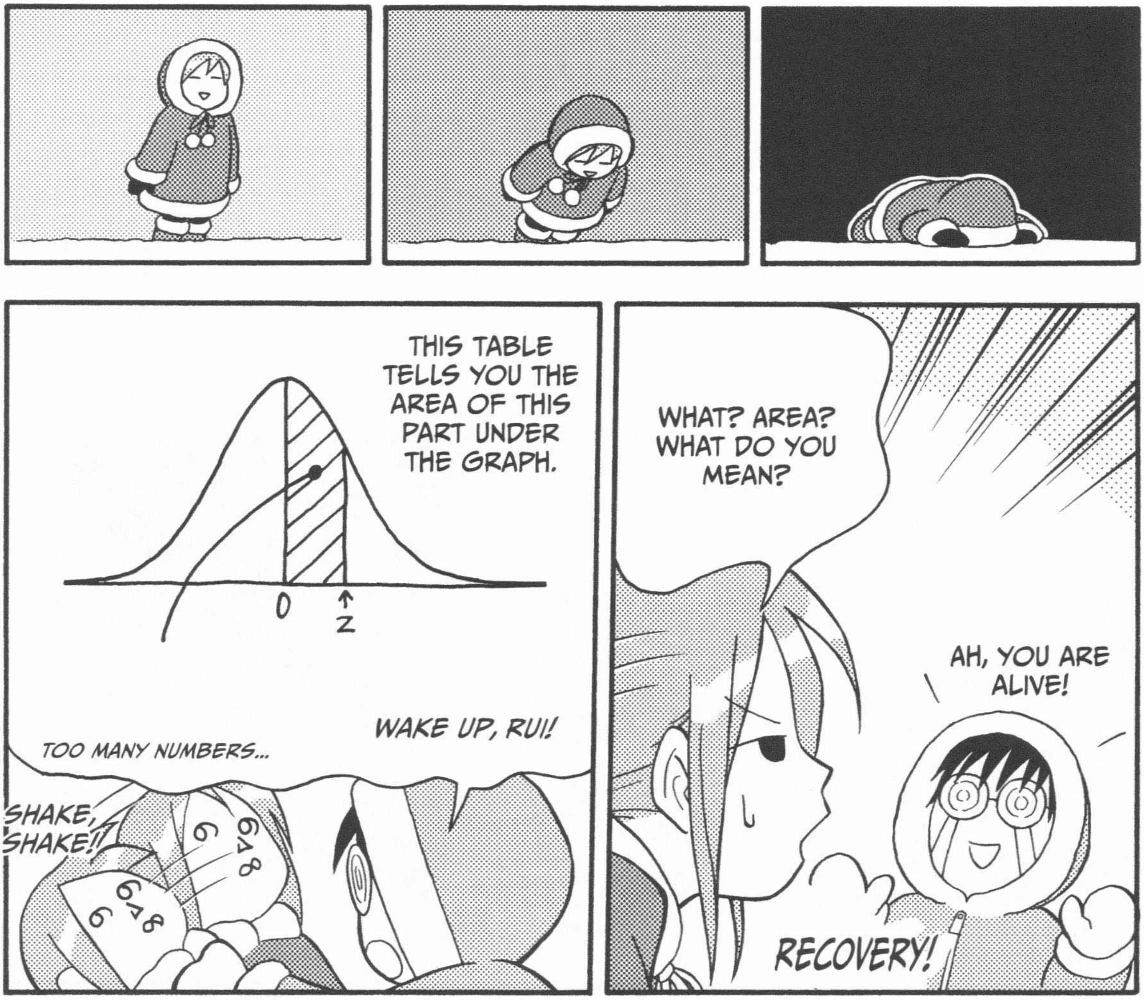
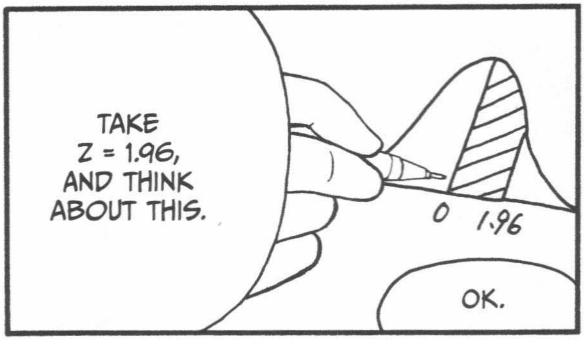
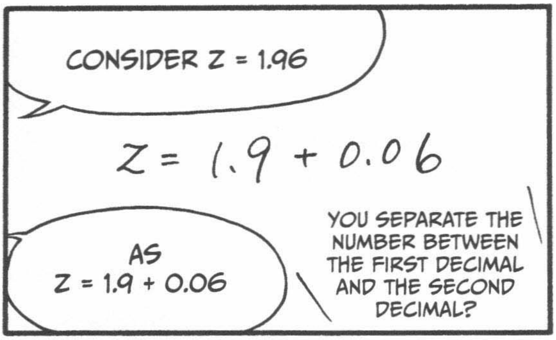


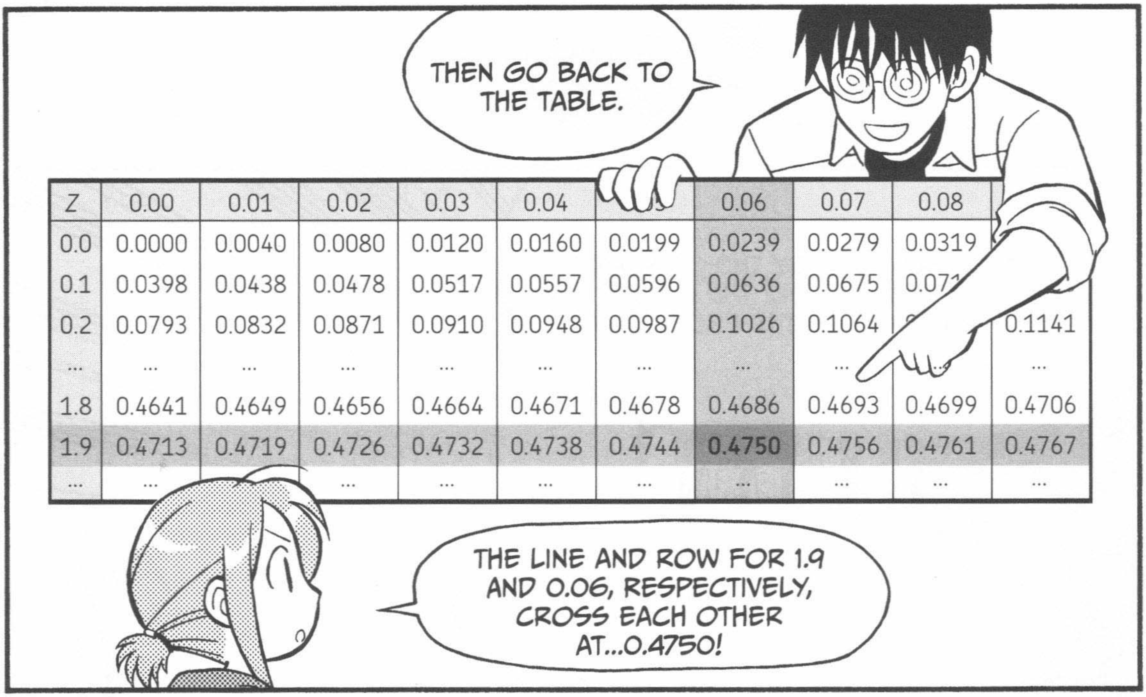
TABLE OF STANDARD NORMAL DISTRIBUTION

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
	…	…	…	…	…	…	…	…	…	…
	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
	…	…	…	…	…	…	…	…	…	…

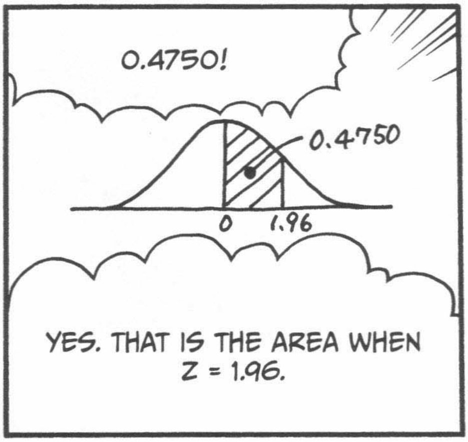


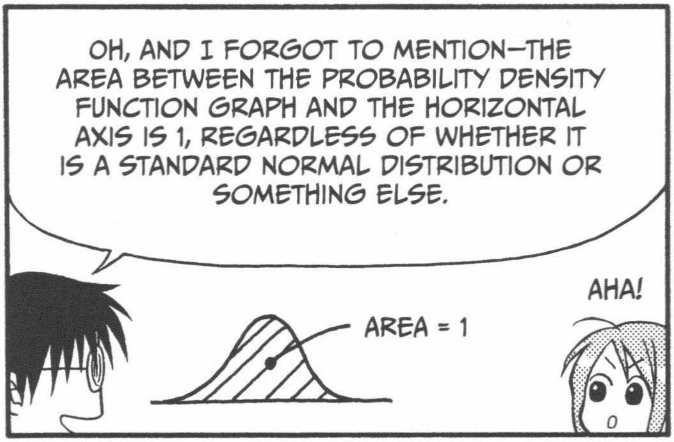


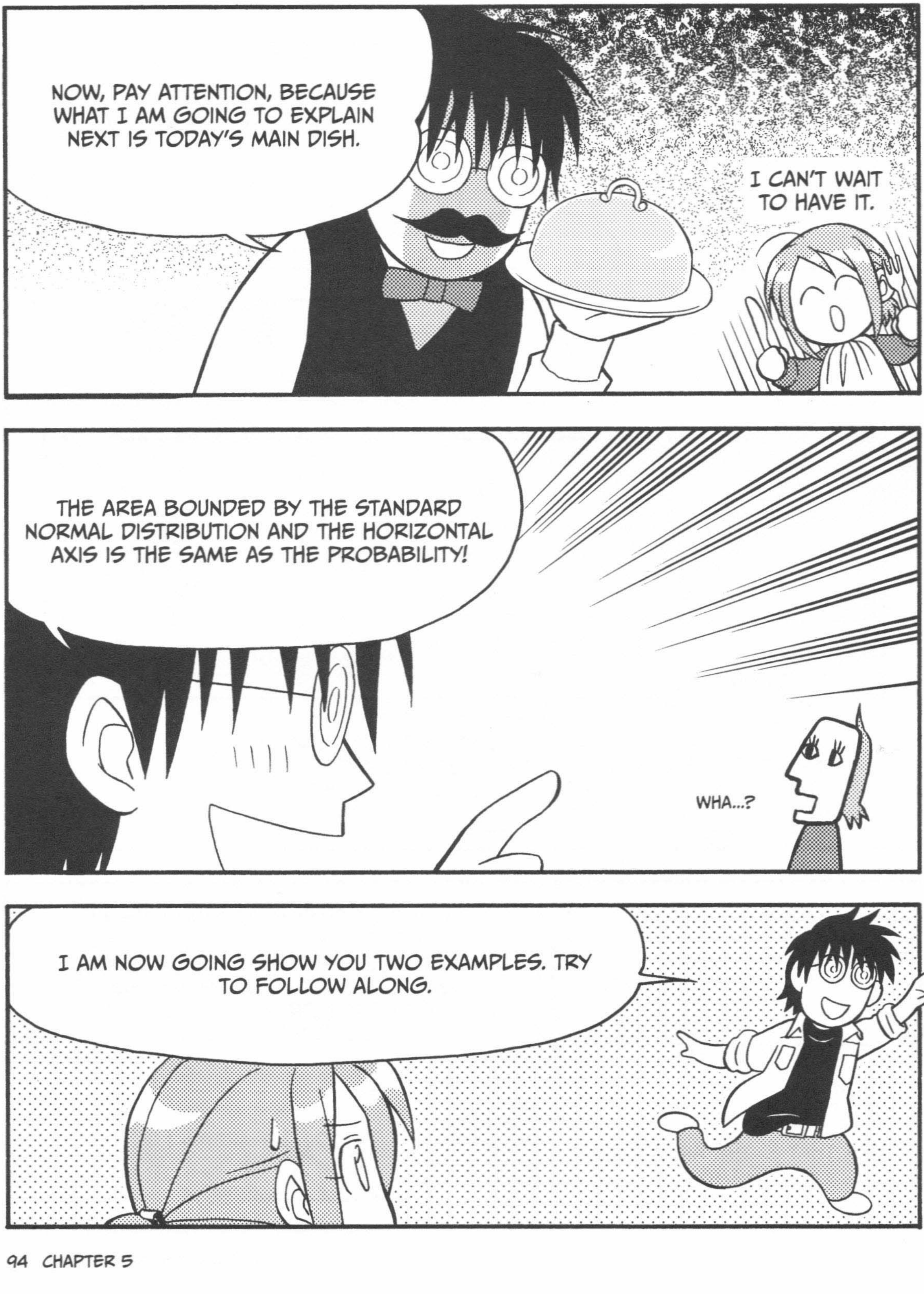




Z	0.00	0.01	0.02	0.03	0.04		0.06	0.07	0.08	
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.071	
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064		0.1141
...	...	...	...	...	...	...	...	...		...
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	**0.4750**	0.4756	0.4761	0.4767
...	...	...	...	...	...	...	...	...	...	...



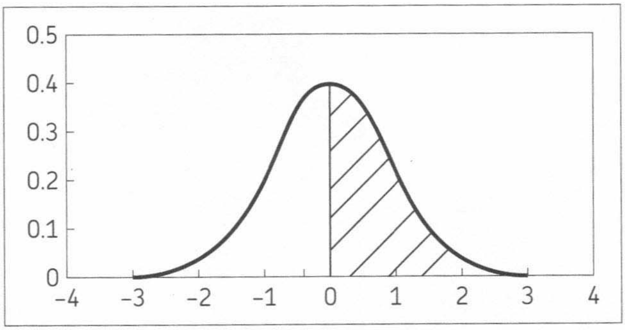



NOW, PAY ATTENTION, BECAUSE WHAT I AM GOING TO EXPLAIN NEXT IS TODAY'S MAIN DISH.
I CAN'T WAIT TO HAVE IT.
THE AREA BOUNDED BY THE STANDARD NORMAL DISTRIBUTION AND THE HORIZONTAL AXIS IS THE SAME AS THE PROBABILITY!
WHA...?
I AM NOW GOING SHOW YOU TWO EXAMPLES. TRY TO FOLLOW ALONG.

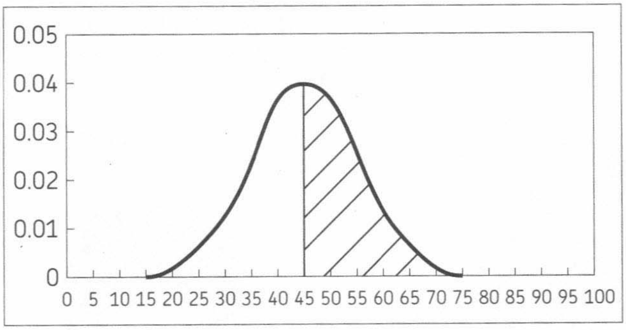
EXAMPLE I

All high school freshmen in prefecture B took a math test. After the tests were marked, the test results turned out to follow a normal distribution with a mean of 45 and a standard deviation of 10. Now, think carefully. The five sentences below all have the same meaning.

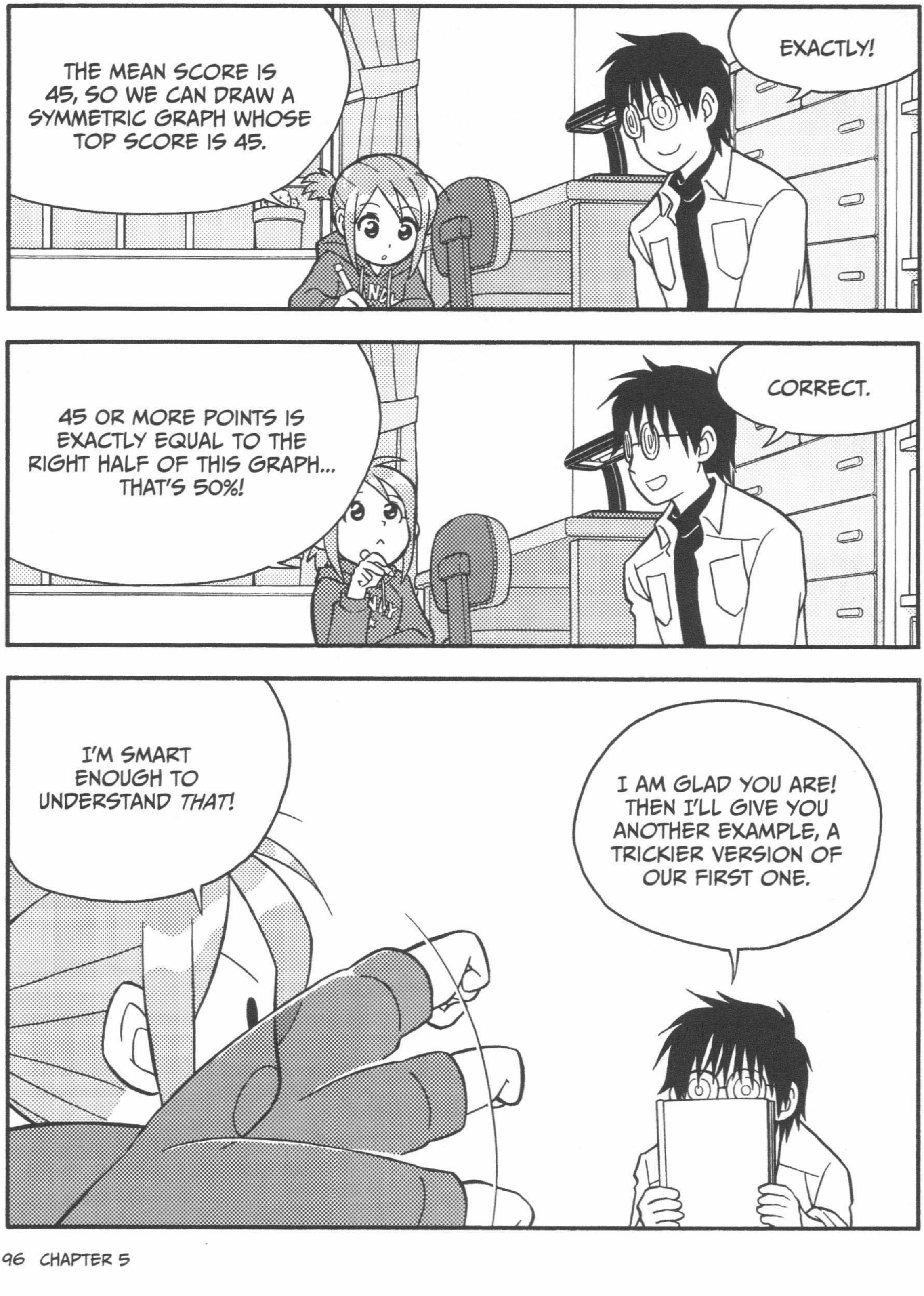
1. In a normal distribution with an average of 45 and a standard deviation of 10, the shaded area in the chart below is 0.5.



2. The ratio of students who scored 45 points or more is 0.5 (50% of all students tested).
3. When one student is randomly chosen from all students tested, the probability that the student's score is 45 or more is 0.5 (50%).
4. In a normal distribution of standardized "math test results," the ratio of students with a standard score of 0 or more is 0.5 (50% of all students tested).



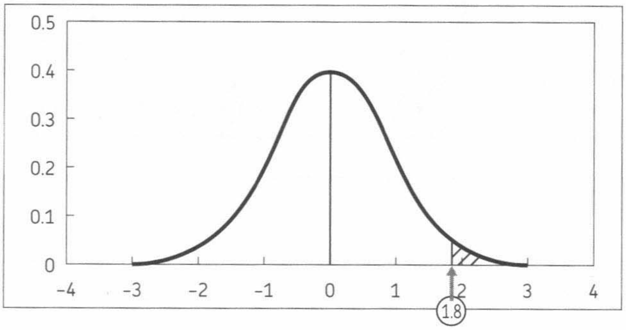
5. When one student's results are randomly chosen from all of those tested in a normal distribution of standardized "math test results," the probability that the selected student's standard score is 0 or more is 0.5 (50%).


THE MEAN SCORE IS 45, SO WE CAN DRAW A SYMMETRIC GRAPH WHOSE TOP SCORE IS 45.
EXACTLY!
45 OR MORE POINTS IS EXACTLY EQUAL TO THE RIGHT HALF OF THIS GRAPH... THAT'S 50%!
CORRECT.
I'M SMART ENOUGH TO UNDERSTAND *THAT*!
I AM GLAD YOU ARE! THEN I'LL GIVE YOU ANOTHER EXAMPLE, A TRICKIER VERSION OF OUR FIRST ONE.

EXAMPLE II

All high school freshmen in prefecture B took a math test. Now, think carefully. The five sentences below all have the same meaning.

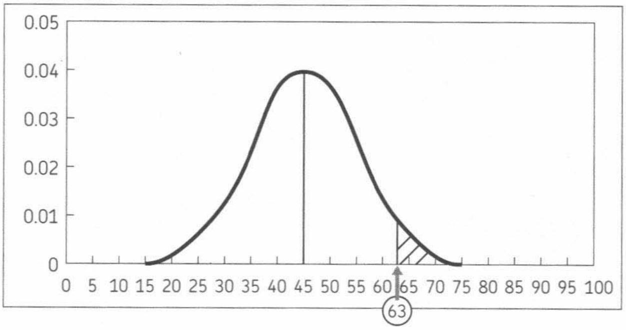
1. In a normal distribution with a mean of 45 and a standard deviation of 10, the shaded area in the chart below is 0.5 - 0.4641 = 0.0359.



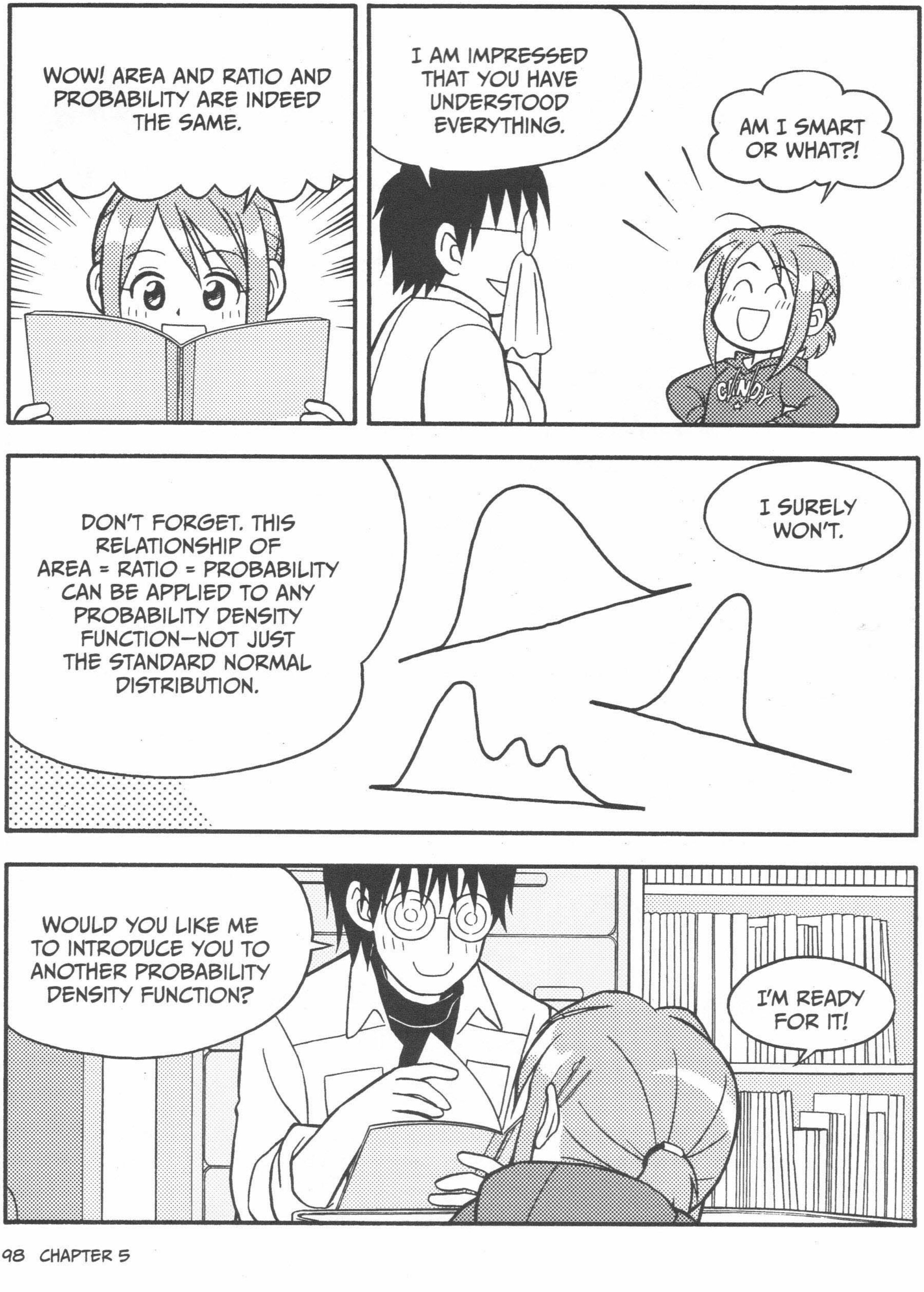
2. The ratio of students who scored 63 points or more is 0.5 – 0.4641 = 0.0359 (3.59% of all students tested).

3. When one student is randomly chosen from all those tested, the probability that the student's score is 63 or more is 0.5 – 0.4641 = 0.0359 (3.59%).

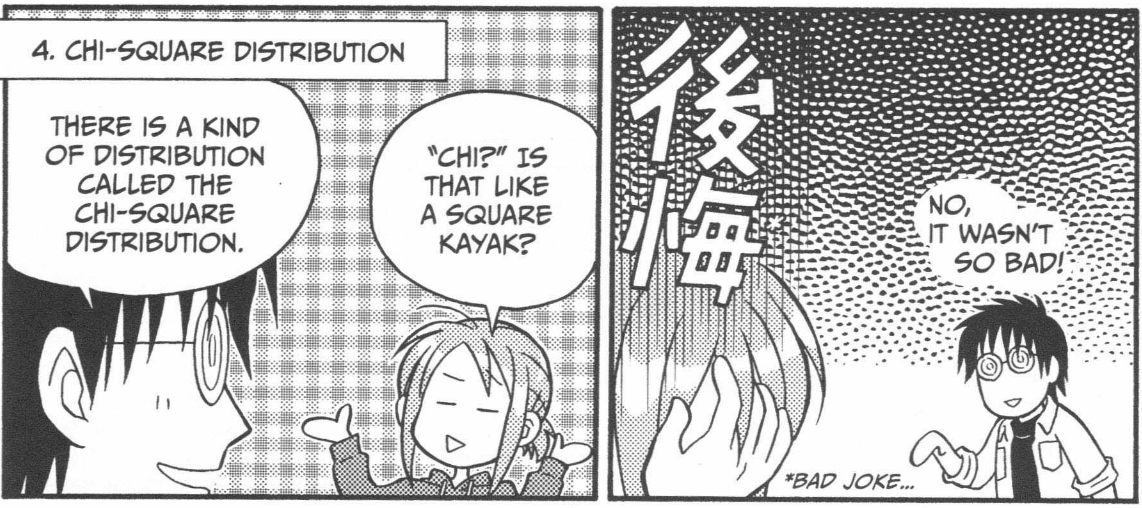
4. In a normal distribution of standardized test results, the ratio of students with standard scores (or z-scores) of 1.8 or more [(each value – average) ÷ standard deviation = (63 – 45) ÷ 10 = 18 ÷ 10 = 1.8] is 3.59% (0.5 - 0.4641 = 0.0359). You can also obtain this value from a table of standard normal distribution.



5. When one student is randomly chosen from all those tested in a normal distribution of standardized "math test results," the probability that the student's standard score is 1.8 or more is 0.5 – 0.4641 = 0.0359 (3.59%).


WOW! AREA AND RATIO AND PROBABILITY ARE INDEED THE SAME.
I AM IMPRESSED THAT YOU HAVE UNDERSTOOD EVERYTHING.
AM I SMART OR WHAT?!
CINDY
DON'T FORGET. THIS RELATIONSHIP OF AREA = RATIO = PROBABILITY CAN BE APPLIED TO ANY PROBABILITY DENSITY FUNCTION—NOT JUST THE STANDARD NORMAL DISTRIBUTION.
I SURELY WON'T.
WOULD YOU LIKE ME TO INTRODUCE YOU TO ANOTHER PROBABILITY DENSITY FUNCTION?
I'M READY FOR IT!

4. CHI-SQUARE DISTRIBUTION



WHEN THE PROBABILITY DENSITY FUNCTION IS...

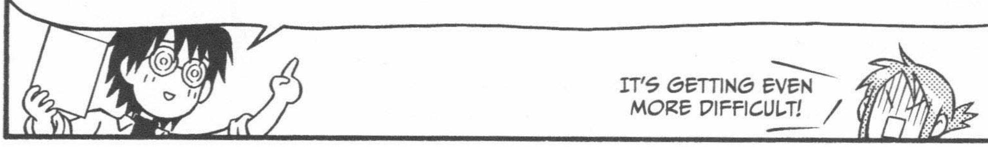
WHEN $x > 0$...

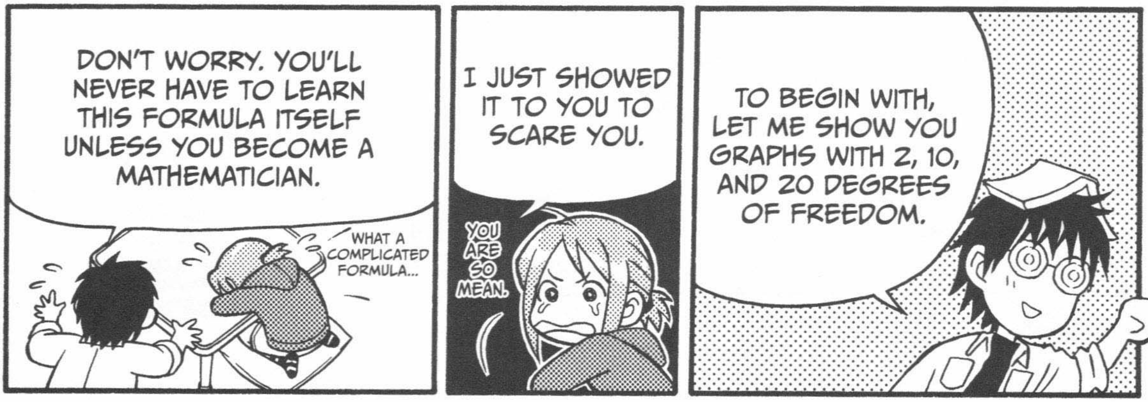
$$f(x) = \frac{1}{2^{\frac{\mathrm{df}}{2}} \times \int_0^\infty x^{\frac{\mathrm{df}}{2}-1} e^{-x} dx} \times x^{\frac{\mathrm{df}}{2}-1} \times e^{-\frac{x}{2}}$$

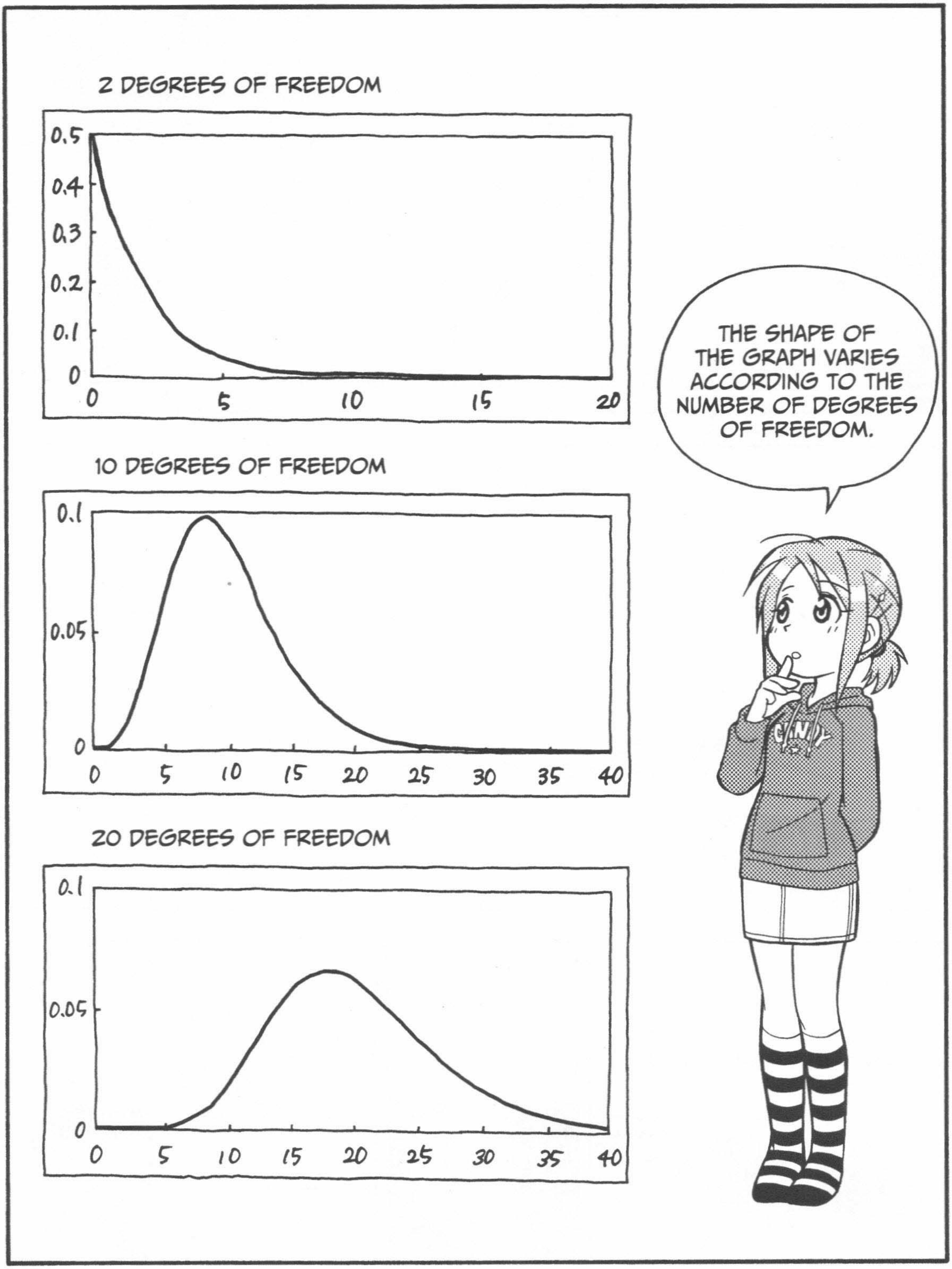
WHEN $x \leq 0$...

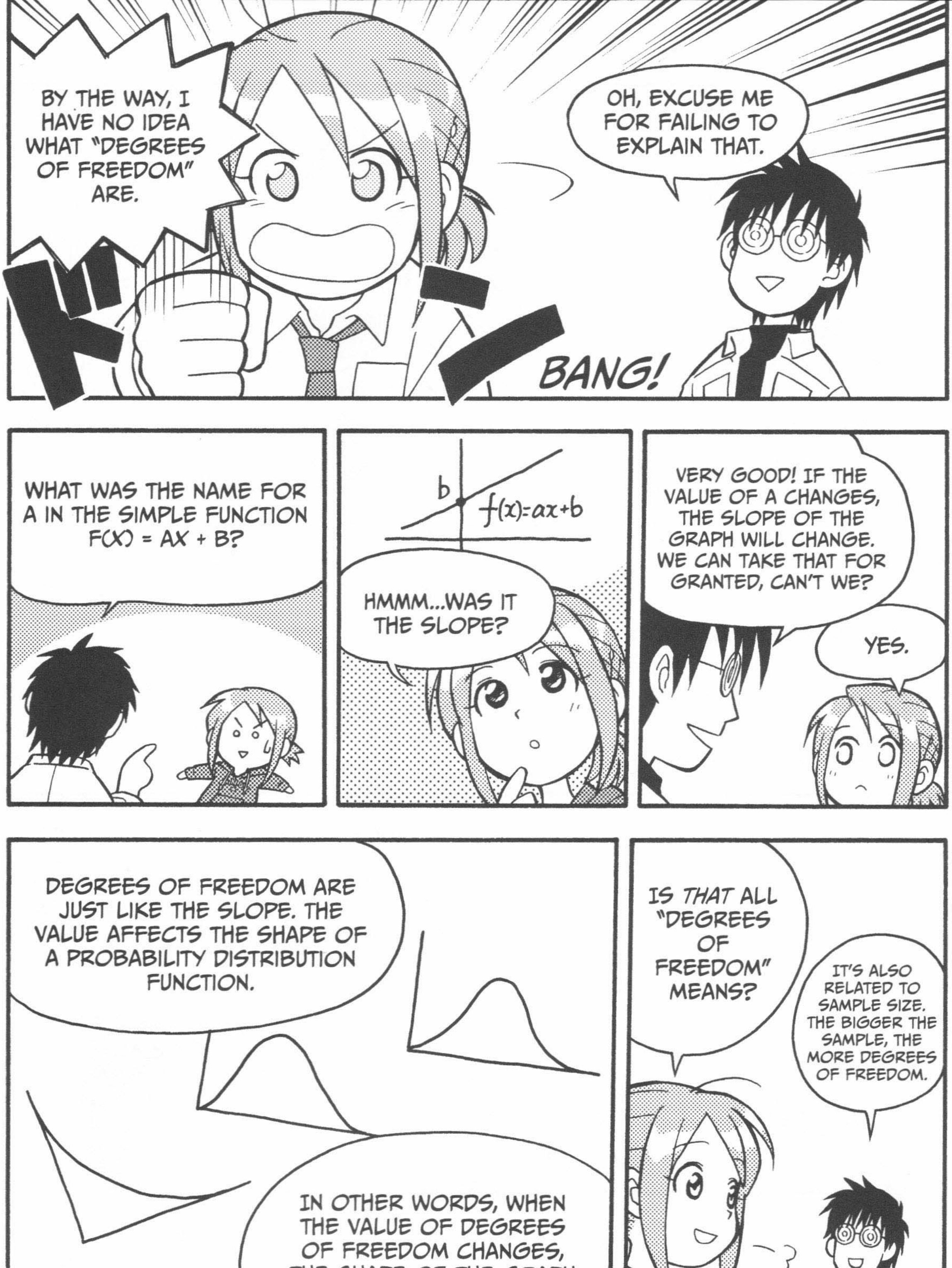
$$f(x) = 0$$

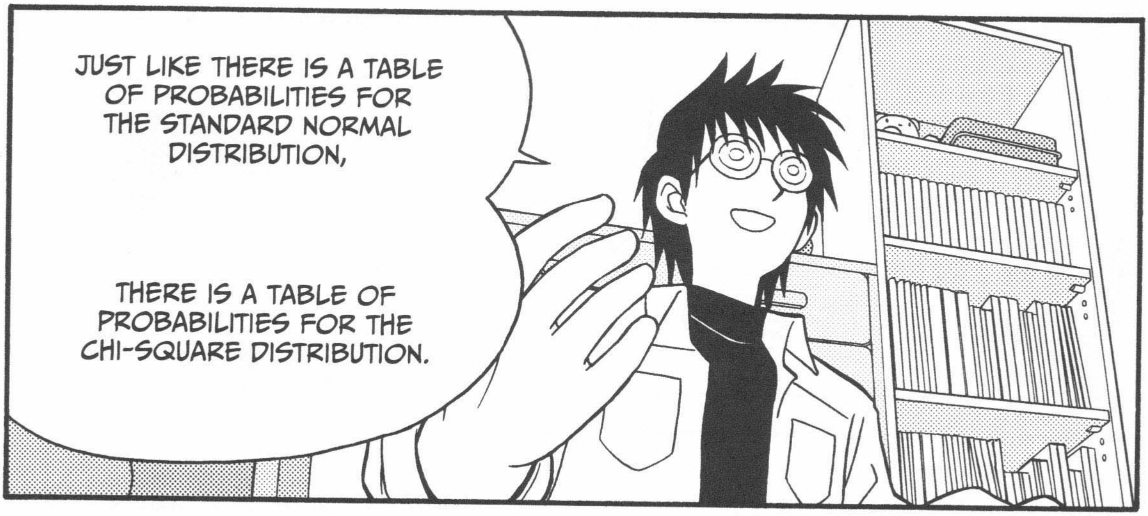
WE SAY, "x FOLLOWS A CHI-SQUARE DISTRIBUTION WITH n DEGREES OF FREEDOM (DF)" IN STATISTICS.

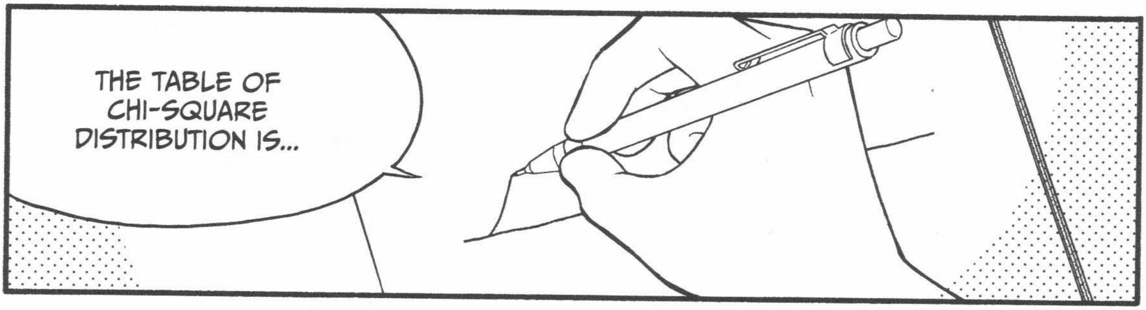


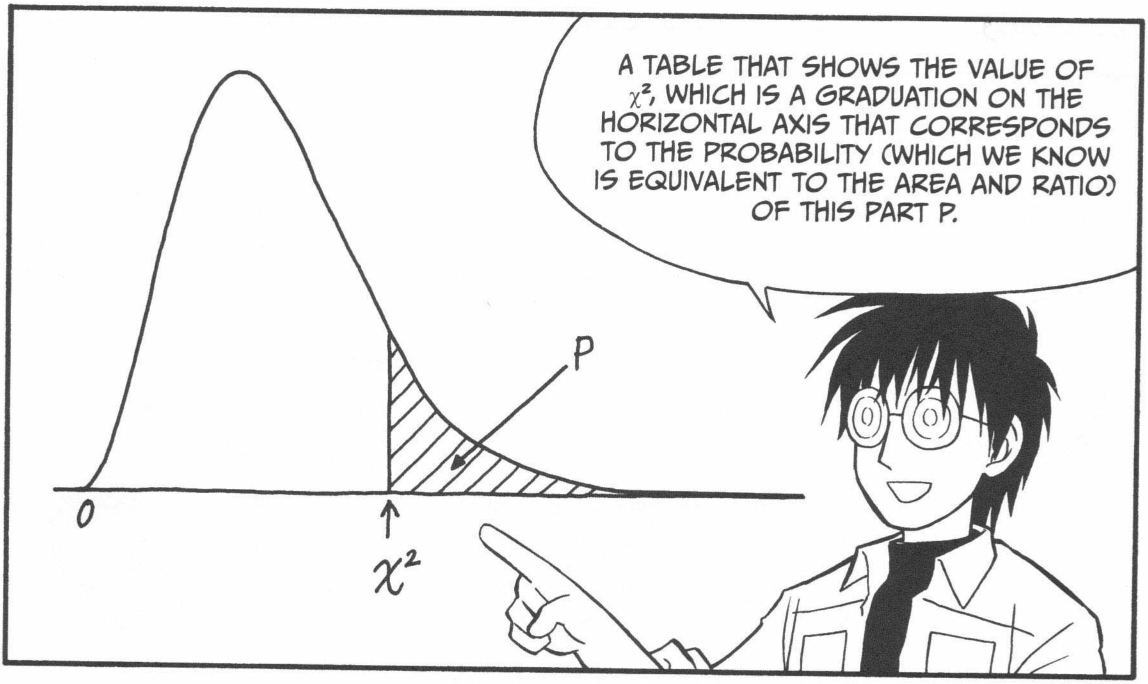


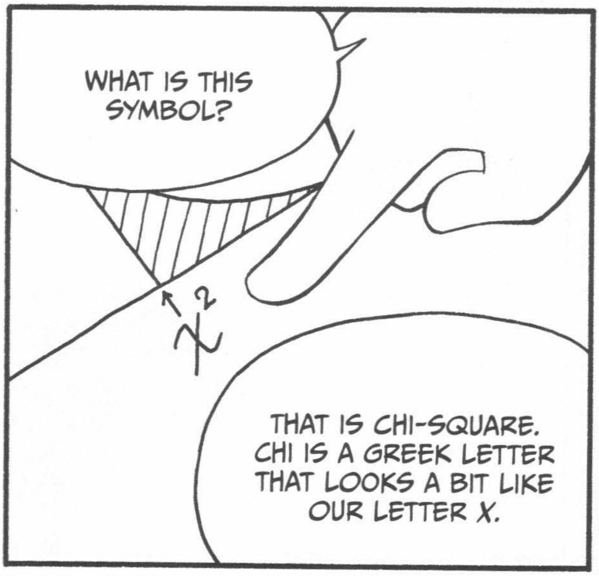

2 DEGREES OF FREEDOM
0.5
0.4
0.3
0.2
0.1
0
0
5
10
15
20
10 DEGREES OF FREEDOM
0.1
0.05
0
0
5
10
15
20
25
30
35
40
20 DEGREES OF FREEDOM
0.1
0.05
0
0
5
10
15
20
25
30
35
40
THE SHAPE OF THE GRAPH VARIES ACCORDING TO THE NUMBER OF DEGREES OF FREEDOM.


BY THE WAY, I HAVE NO IDEA WHAT "DEGREES OF FREEDOM" ARE.
OH, EXCUSE ME FOR FAILING TO EXPLAIN THAT.
BANG!
WHAT WAS THE NAME FOR A IN THE SIMPLE FUNCTION F(X) = AX + B?
b
f(x)=ax+b
HMMM...WAS IT THE SLOPE?
VERY GOOD! IF THE VALUE OF A CHANGES, THE SLOPE OF THE GRAPH WILL CHANGE. WE CAN TAKE THAT FOR GRANTED, CAN'T WE?
YES.
DEGREES OF FREEDOM ARE JUST LIKE THE SLOPE. THE VALUE AFFECTS THE SHAPE OF A PROBABILITY DISTRIBUTION FUNCTION.
IN OTHER WORDS, WHEN THE VALUE OF DEGREES OF FREEDOM CHANGES, THE SHAPE OF THE GRAPH CHANGES AS WELL.
IS THAT ALL "DEGREES OF FREEDOM" MEANS?
IT'S ALSO RELATED TO SAMPLE SIZE. THE BIGGER THE SAMPLE, THE MORE DEGREES OF FREEDOM.


JUST LIKE THERE IS A TABLE OF PROBABILITIES FOR THE STANDARD NORMAL DISTRIBUTION,
THERE IS A TABLE OF PROBABILITIES FOR THE CHI-SQUARE DISTRIBUTION.


THE TABLE OF CHI-SQUARE DISTRIBUTION IS...

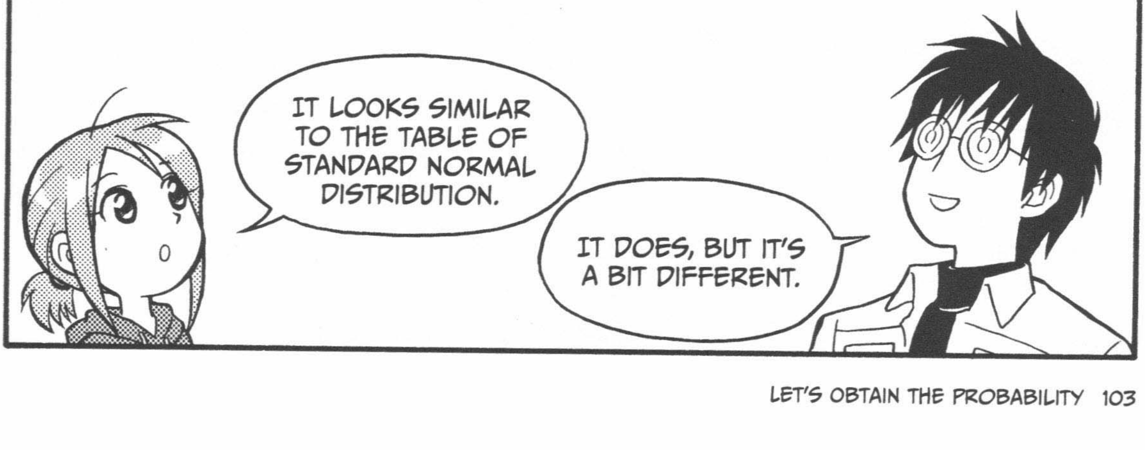

A TABLE THAT SHOWS THE VALUE OF χ^2, WHICH IS A GRADUATION ON THE HORIZONTAL AXIS THAT CORRESPONDS TO THE PROBABILITY (WHICH WE KNOW IS EQUIVALENT TO THE AREA AND RATIO) OF THIS PART P.
P
0
χ^2

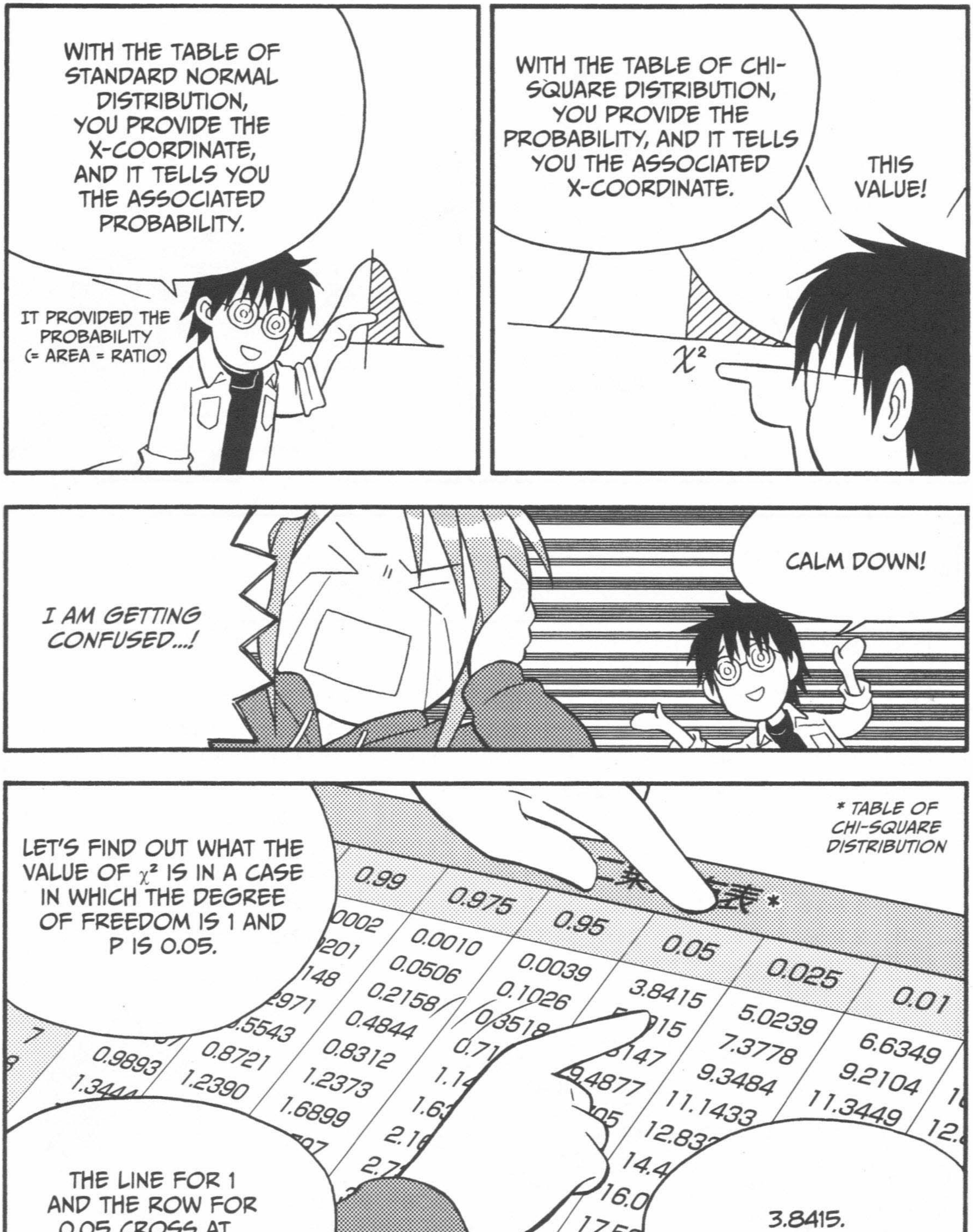


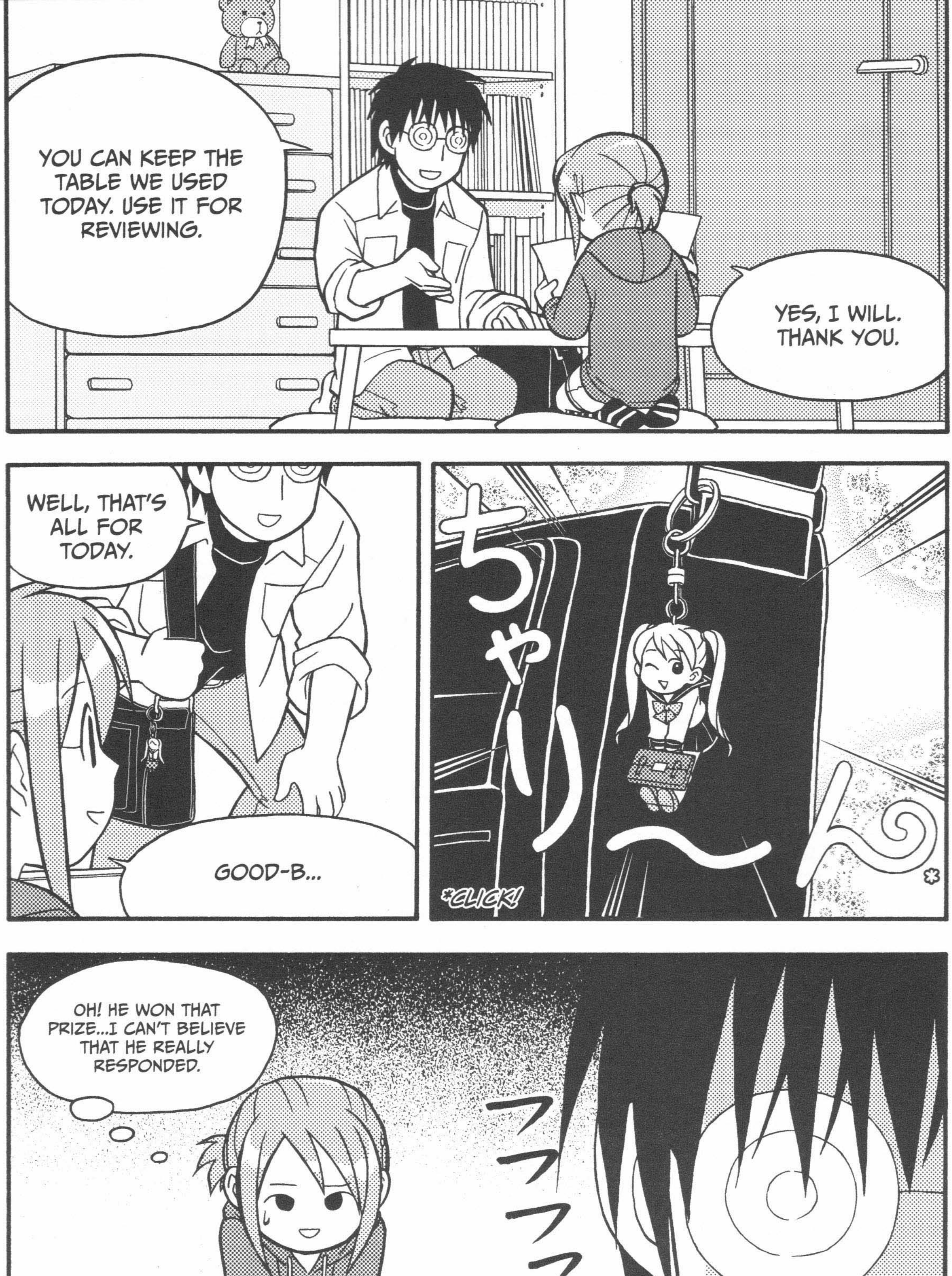
DEGREES OF FREEDOM

TABLE OF CHI-SQUARE DISTRIBUTION

P	0.995	0.99	0.975	0.95	0.05	0.025	0.01	0.005
1	0.000039	0.0002	0.0010	0.0039	3.8415	5.0239	6.6349	7.8794
2	0.0100	0.0201	0.0506	0.1026	5.9915	7.3778	9.2104	10.5965
3	0.0717	0.1148	0.2158	0.3518	7.8147	9.3484	11.3449	12.8381
4	0.2070	0.2971	0.4844	0.7107	9.4877	11.1433	13.2767	14.8602
5	0.4118	0.5543	0.8312	1.1455	11.0705	12.8325	15.0863	16.7496
6	0.6757	0.8721	1.2373	1.6354	12.5916	14.4494	16.8119	18.5475
7	0.9893	1.2390	1.6899	2.1673	14.0671	16.0128	18.4753	20.2777
8	1.3444	1.6465	2.1797	2.7326	15.5073	17.5345	20.0902	21.9549
9	1.7349	2.0879	2.7004	3.3251	16.9190	19.0228	21.6660	23.5893
10	2.1558	2.5582	3.2470	3.9403	18.3070	20.4832	23.2093	25.1881
...	...	...	...	...	...	...	...	...




WITH THE TABLE OF STANDARD NORMAL DISTRIBUTION, YOU PROVIDE THE X-COORDINATE, AND IT TELLS YOU THE ASSOCIATED PROBABILITY.
IT PROVIDED THE PROBABILITY (= AREA = RATIO)
WITH THE TABLE OF CHI-SQUARE DISTRIBUTION, YOU PROVIDE THE PROBABILITY, AND IT TELLS YOU THE ASSOCIATED X-COORDINATE.
THIS VALUE!
χ^2
I AM GETTING CONFUSED...!
CALM DOWN!
LET'S FIND OUT WHAT THE VALUE OF χ^2 IS IN A CASE IN WHICH THE DEGREE OF FREEDOM IS 1 AND P IS 0.05.
* TABLE OF CHI-SQUARE DISTRIBUTION
0.99 0.975 0.95 0.05 0.025 0.01
THE LINE FOR 1 AND THE ROW FOR 0.05 CROSS AT...
3.8415.


YOU CAN KEEP THE TABLE WE USED TODAY. USE IT FOR REVIEWING.
YES, I WILL. THANK YOU.
WELL, THAT'S ALL FOR TODAY.
GOOD-B...
ちゃり～ん*
*CLICK!
OH! HE WON THAT PRIZE...I CAN'T BELIEVE THAT HE REALLY RESPONDED.
フフフ…

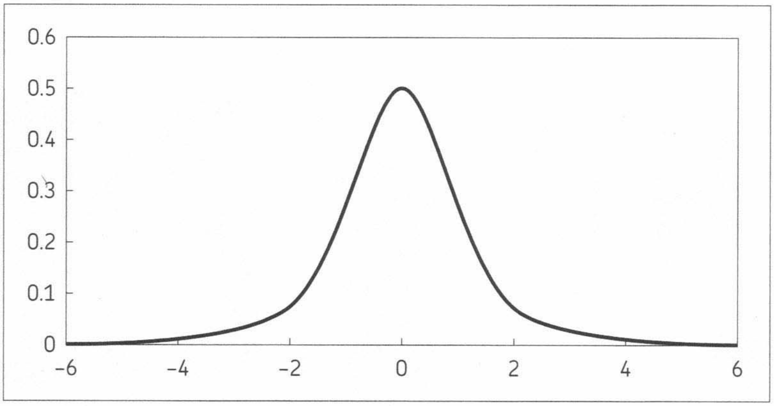
5. T DISTRIBUTION

The probability density function below is a popular topic in statistics.

$$f(x) = \frac{\int_0^\infty x^{\frac{\text{df}+1}{2}-1} e^{-x}dx}{\sqrt{\text{df}\times\pi}\times\int_0^\infty x^{\frac{\text{df}}{2}-1} e^{-x}dx} \times \left(1+\frac{x^2}{\text{df}}\right)^{-\frac{\text{df}+1}{2}}$$

When the probability density function for *x* looks like this, we say, "*x* follows a t distribution with *n* degrees of freedom."

Here is a case with 5 degrees of freedom:



6. F DISTRIBUTION

The probability density function below is a popular topic in statistics.

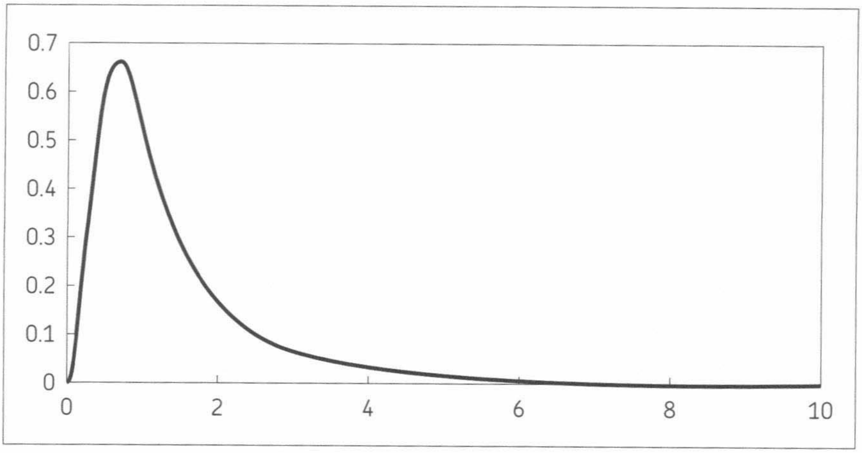
when $x > 0$:

$$f(x) = \frac{\left(\int_0^\infty x^{\frac{\left(\text{first df}+\text{second df}\right)}{2}-1} e^{-x}dx\right)\times(\text{first df})^{\frac{\text{first df}}{2}}\times(\text{second df})^{\frac{\text{second df}}{2}}}{\left(\int_0^\infty x^{\frac{\text{first df}}{2}-1} e^{-x}dx\right)\times\left(\int_0^\infty x^{\frac{\text{second df}}{2}-1} e^{-x}dx\right)} \times \frac{x^{\frac{\text{first df}}{2}-1}}{(\text{first df}\times x+\text{second df})^{\frac{\left(\text{first df}+\text{second df}\right)}{2}}}$$

when $x \le 0$: $f(x) = 0$

When the probability density function for *x* looks like this, we say, "*x* follows an F distribution with the first degree of freedom *m* and the second degree of freedom *n*."

Here is a case in which the first degree of freedom is 10 and the second degree of freedom is 5:



7. DISTRIBUTIONS AND EXCEL

Until the rise of personal computers (roughly speaking, around the beginning of the 1990s), it was difficult for an individual to calculate the probability without tables of standard normal distribution or chi-square distribution. However, these tables of distribution are not used much anymore—you can use Excel functions to find the same values as the ones provided by the tables. This enables individuals to calculate even more types of values than the ones found in the tables of distribution. Table 5-1 summarizes Excel functions related to various distributions. (Refer to the appendix on page 191 for more information on making calculations with Excel.)

TABLE 5-1: EXCEL FUNCTIONS RELATED TO VARIOUS DISTRIBUTIONS

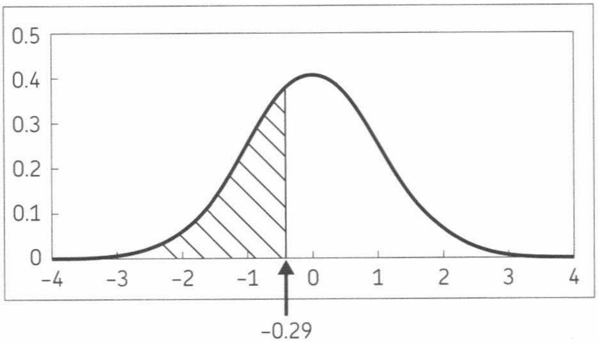
Distribution	Functions	Feature of the function
normal*	NORMDIST	Calculates the probability that corresponds to a point on the horizontal axis.
normal	NORMINV	Calculates a point on the horizontal axis that corresponds to the probability.
standard normal	NORMSDIST	Calculates the probability that corresponds to a point on the horizontal axis.
standard normal	NORMSINV	Calculates a point on the horizontal axis that corresponds to the probability.
chi-square	CHIDIST	Calculates the probability that corresponds to a point on the horizontal axis.
chi-square	CHIINV	Calculates a point on the horizontal axis that corresponds to the probability.
t	TDIST	Calculates the probability that corresponds to a point on the horizontal axis.
t	TINV	Calculates a point on the horizontal axis that corresponds to the probability.
F	FDIST	Calculates the probability that corresponds to a point on the horizontal axis.
F	FINV	Calculates a point on the horizontal axis that corresponds to the probability.

* The probability density function for normal distribution is affected by the mean and standard deviation. Thus, it is impossible to make a "table of normal distribution," and no such thing exists in this world. However, by using Excel, you can conveniently calculate the values and make a table relevant to the normal distribution.

EXERCISE AND ANSWER

EXERCISE

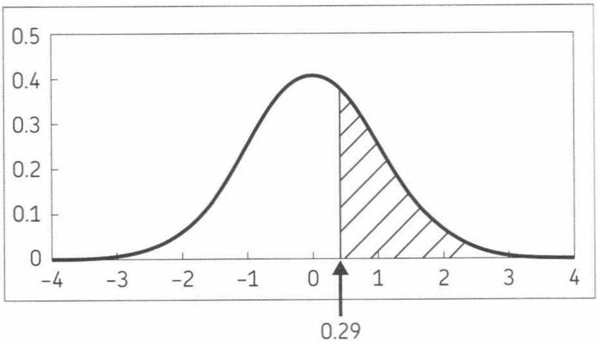
1. Calculate the probability (the shaded area in the graph below) using the table of standard normal distribution on page 93.



2. Calculate the value of χ^2 when there are 2 degrees of freedom and P is 0.05 using the table of chi-square distribution on page 103.

ANSWER

1. Because the standard normal distribution is symmetrical, the probability in question is equal to the probability shown in the graph below.



The probability when z = 0.29 = 0.2 + 0.09 is 0.1141 according to the table of standard normal distribution. Therefore, the probability to be obtained is 0.5 - 0.1141 = 0.3859.

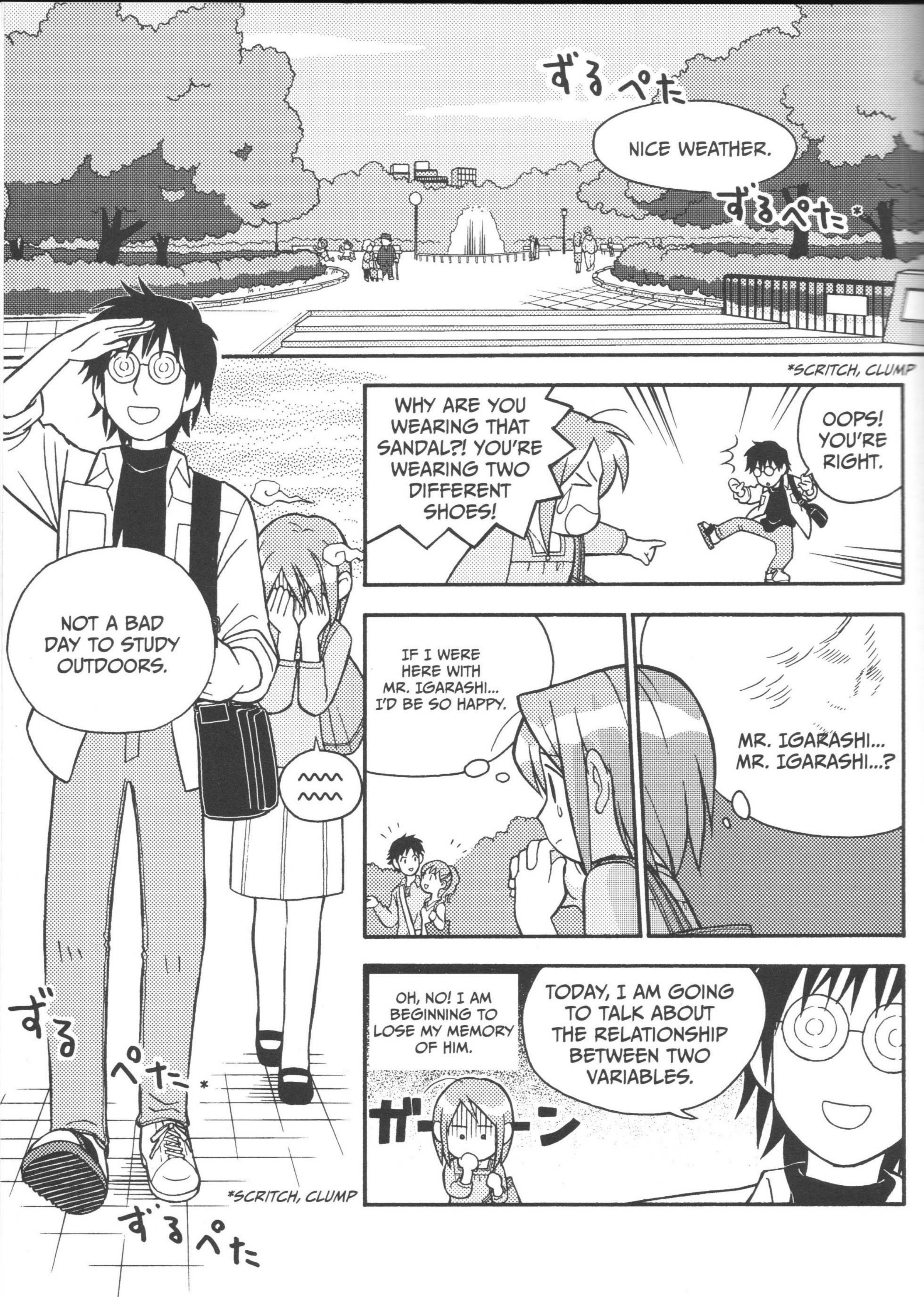
2. The value of χ^2 to be obtained is 5.9915 according to the table of chi-square distribution.

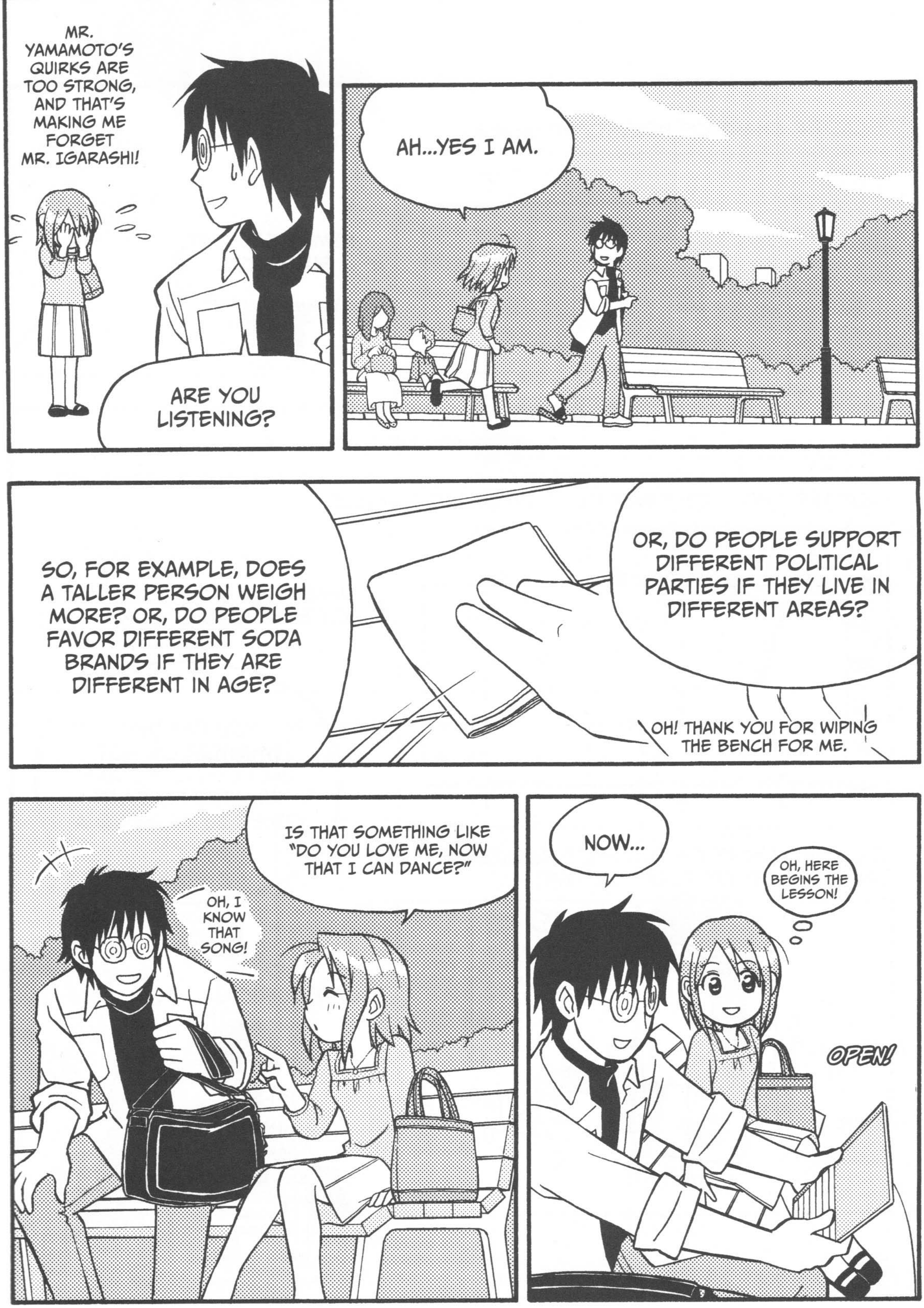
SUMMARY

- Some of the most common probability density functions are:
    - Normal distribution
    - Standard normal distribution
    - Chi-square distribution
    - t distribution
    - F distribution
- The area between the probability density function and the horizontal axis is 1. This area is equivalent to a ratio or a probability.
- By using an Excel function or a table of probabilities for the appropriate distribution, you can calculate:
    - The probability that corresponds to a point on the horizontal axis
    - The point on the horizontal axis that corresponds to the probability

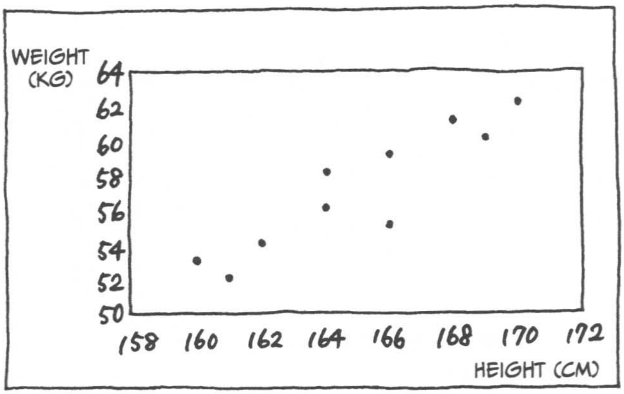
6

LET'S LOOK AT THE RELATIONSHIP BETWEEN TWO VARIABLES


ずるぺた
NICE WEATHER.
ずるぺた*
*SCRITCH, CLUMP
NOT A BAD DAY TO STUDY OUTDOORS.
~~~
~~~
ずる
ぺた*
*SCRITCH, CLUMP
ずるぺた
WHY ARE YOU WEARING THAT SANDAL?! YOU'RE WEARING TWO DIFFERENT SHOES!
OOPS! YOU'RE RIGHT.
IF I WERE HERE WITH MR. IGARASHI... I'D BE SO HAPPY.
MR. IGARASHI... MR. IGARASHI...?
TODAY, I AM GOING TO TALK ABOUT THE RELATIONSHIP BETWEEN TWO VARIABLES.
OH, NO! I AM BEGINNING TO LOSE MY MEMORY OF HIM.
ガーーン

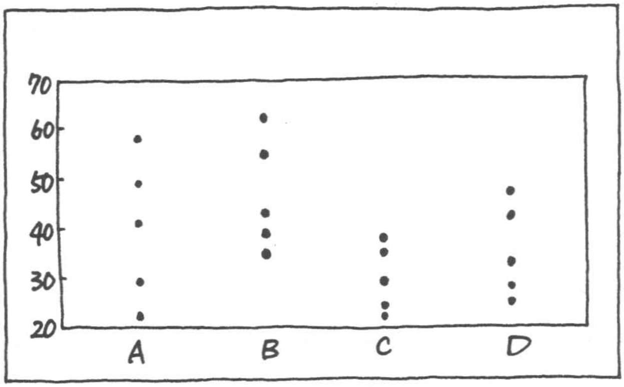

MR. YAMAMOTO'S QUIRKS ARE TOO STRONG, AND THAT'S MAKING ME FORGET MR. IGARASHI!
ARE YOU LISTENING?
AH...YES I AM.
SO, FOR EXAMPLE, DOES A TALLER PERSON WEIGH MORE? OR, DO PEOPLE FAVOR DIFFERENT SODA BRANDS IF THEY ARE DIFFERENT IN AGE?
OR, DO PEOPLE SUPPORT DIFFERENT POLITICAL PARTIES IF THEY LIVE IN DIFFERENT AREAS?
OH! THANK YOU FOR WIPING THE BENCH FOR ME.
IS THAT SOMETHING LIKE "DO YOU LOVE ME, NOW THAT I CAN DANCE?"
OH, I KNOW THAT SONG!
NOW...
OH, HERE BEGINS THE LESSON!
OPEN!

SCATTER CHART OF HEIGHT AND WEIGHT



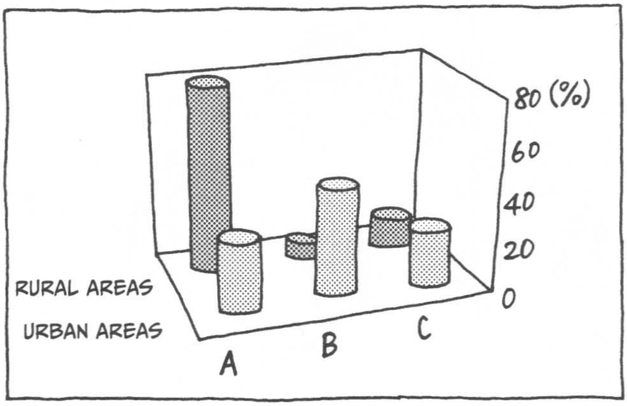
NUMERICAL AND NUMERICAL

SCATTER CHART OF FAVORITE SODA BRAND AND AGE

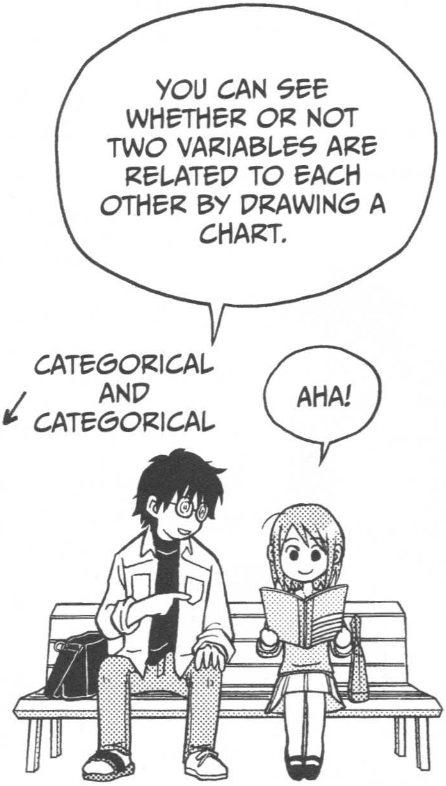


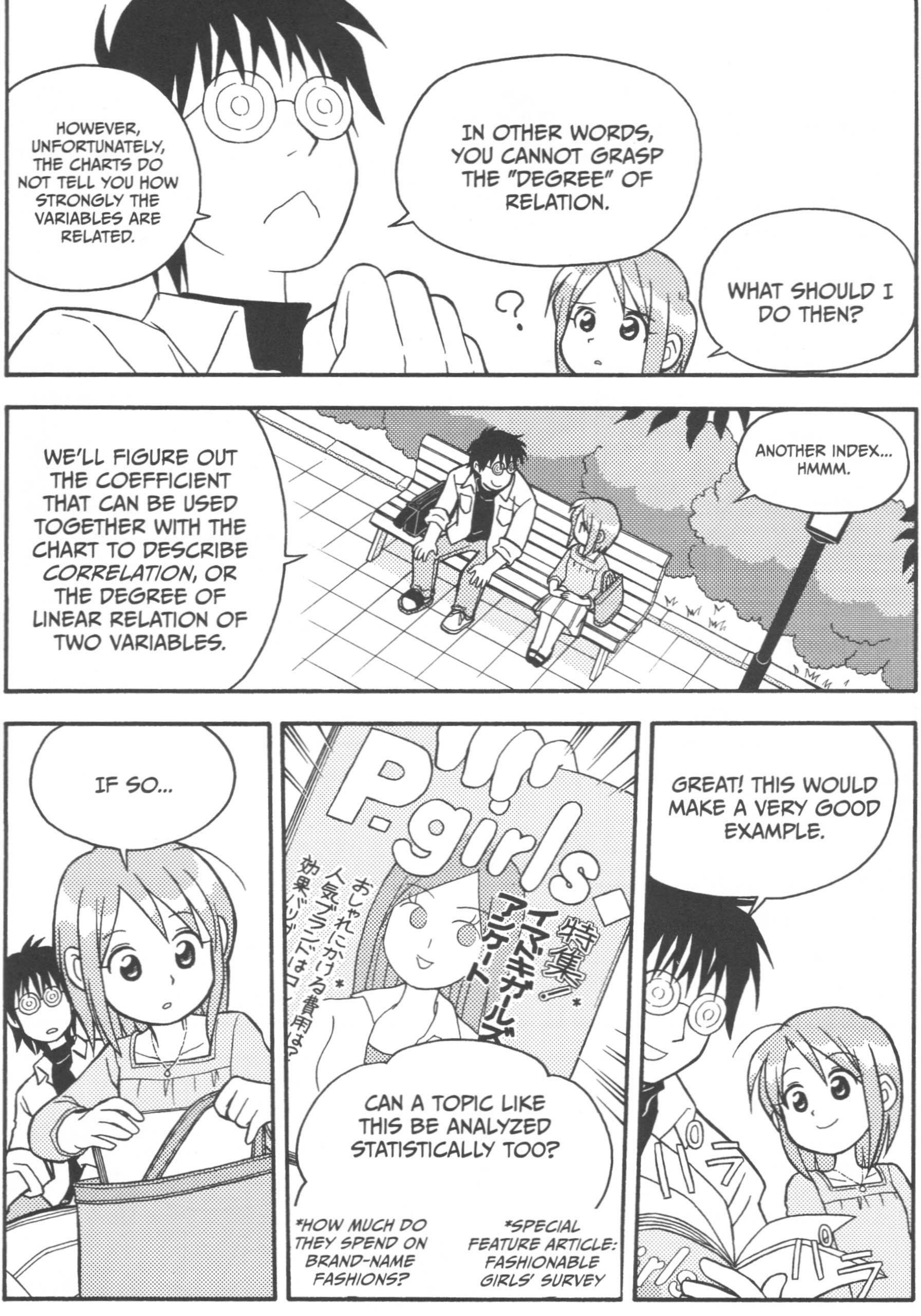
NUMERICAL AND CATEGORICAL

CYLINDER CHART OF PLACE OF RESIDENCE AND SUPPORT OF POLITICAL PARTY X

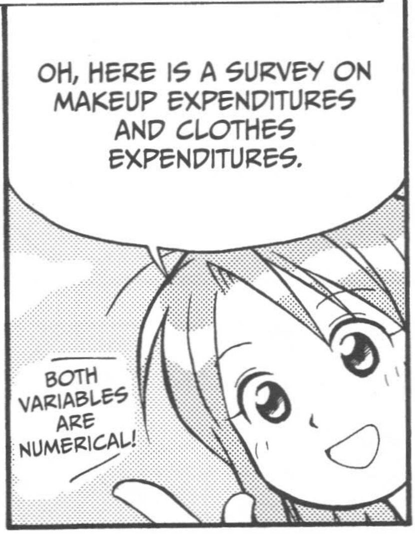


CATEGORICAL AND CATEGORICAL




HOWEVER, UNFORTUNATELY, THE CHARTS DO NOT TELL YOU HOW STRONGLY THE VARIABLES ARE RELATED.
IN OTHER WORDS, YOU CANNOT GRASP THE "DEGREE" OF RELATION.
?
WHAT SHOULD I DO THEN?
WE'LL FIGURE OUT THE COEFFICIENT THAT CAN BE USED TOGETHER WITH THE CHART TO DESCRIBE *CORRELATION*, OR THE DEGREE OF LINEAR RELATION OF TWO VARIABLES.
ANOTHER INDEX... HMMM.
IF SO...
P. girls
特集！イマドキガールズアンケート*
おしゃれにかける費用は*
人気ブランドは
CAN A TOPIC LIKE THIS BE ANALYZED STATISTICALLY TOO?
*HOW MUCH DO THEY SPEND ON BRAND-NAME FASHIONS?
*SPECIAL FEATURE ARTICLE: FASHIONABLE GIRLS' SURVEY
GREAT! THIS WOULD MAKE A VERY GOOD EXAMPLE.

1. CORRELATION COEFFICIENT

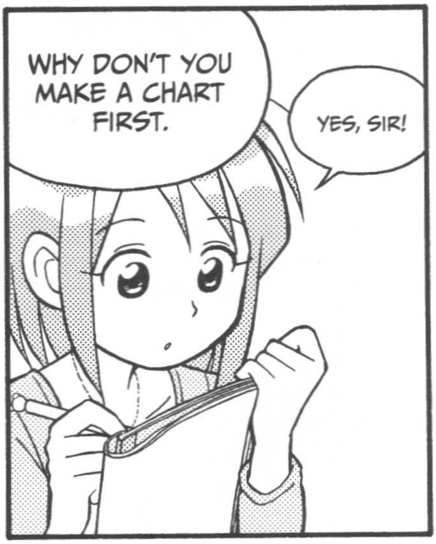


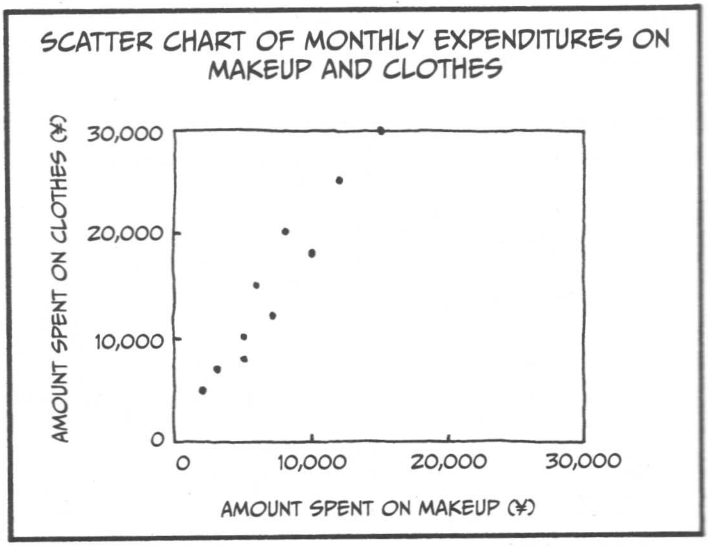
Street survey!

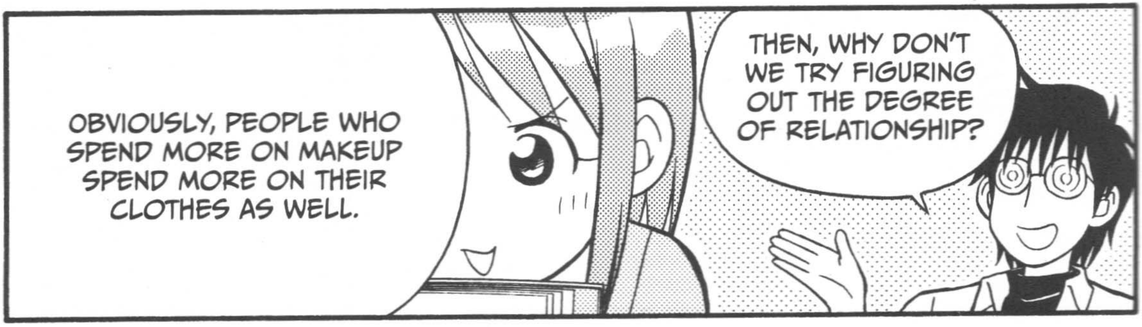
Ten ladies in their 20s answered

Monthly Expenditures on Makeup and Clothes

Respondent	Amount spent on makeup (¥)	Amount spent on clothes (¥)
Ms. A	3,000	7,000
Ms. B	5,000	8,000
Ms. C	12,000	25,000
Ms. D	2,000	5,000
Ms. E	7,000	12,000
Ms. F	15,000	30,000
Ms. G	5,000	10,000
Ms. H	6,000	15,000
Ms. I	8,000	20,000
Ms. J	10,000	18,000

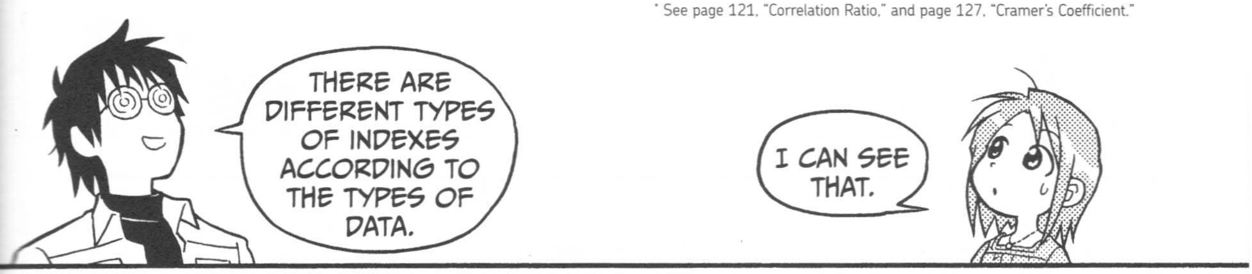


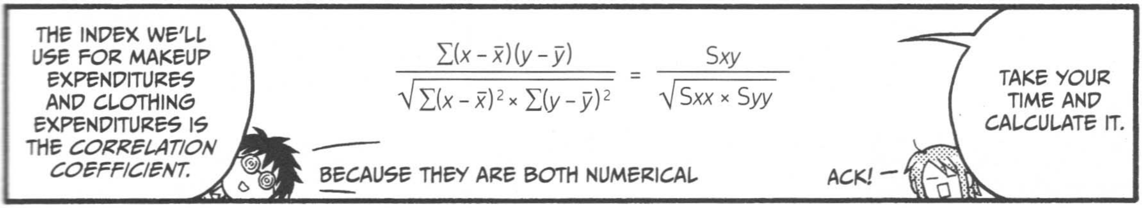




Data types	Index	Value range	Formula
Numerical and numerical	Correlation coefficient	-1 – 1	$\dfrac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \times \sum(y-\bar{y})^2}} = \dfrac{Sxy}{\sqrt{Sxx \times Syy}}$
Numerical and categorical	Correlation ratio*	0 – 1	$\dfrac{\text{interclass variance}}{\text{intraclass variance + interclass variance}}$
Categorical and categorical	Cramer's coefficient*	0 – 1	$\sqrt{\dfrac{\chi_0^2}{\text{the total number of values} \times (\min\{\text{the number of lines in the cross tabulation, the number of rows in the cross tabulation}\} - 1)}}$

* See page 121, "Correlation Ratio," and page 127, "Cramer's Coefficient."



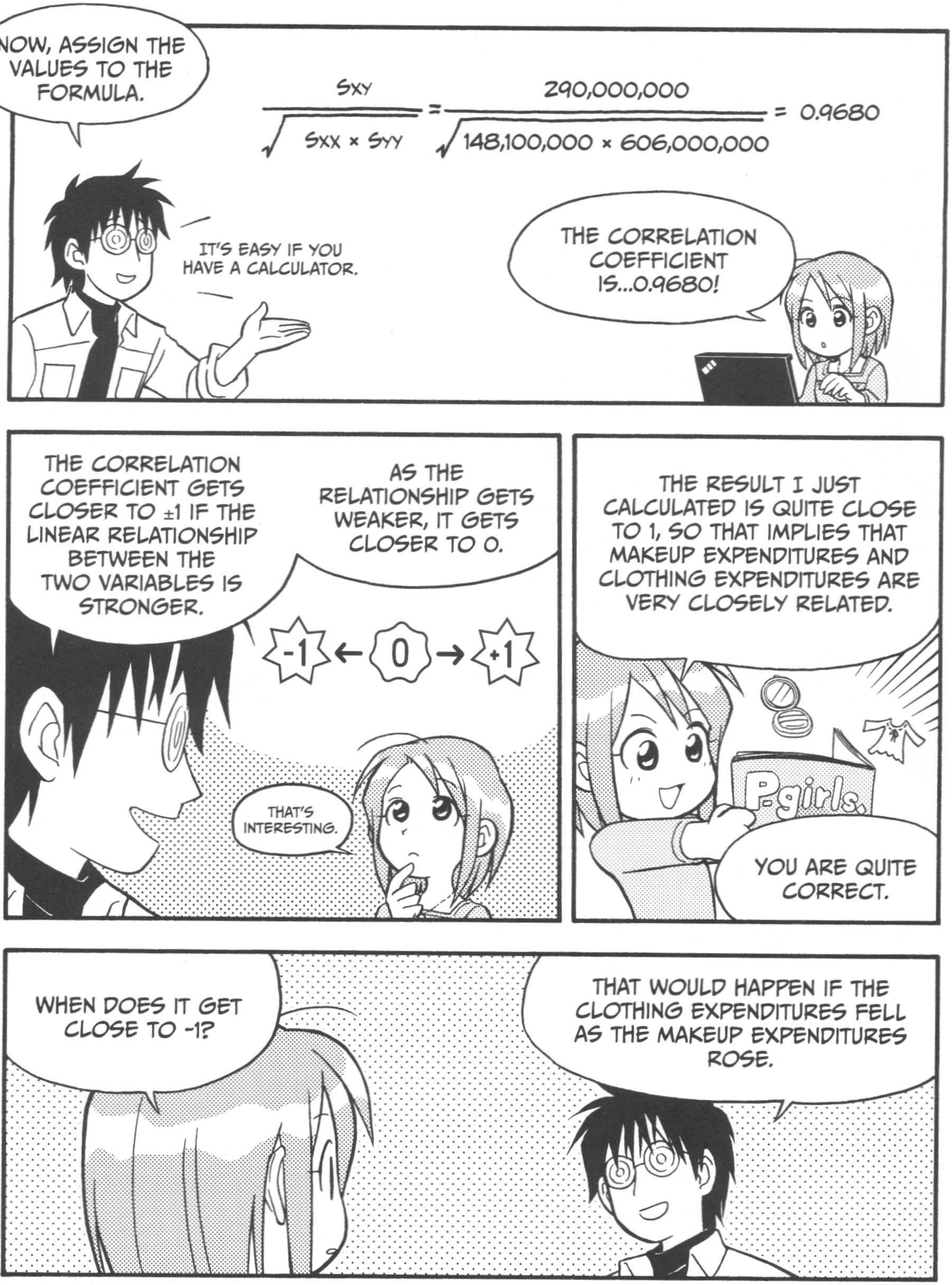


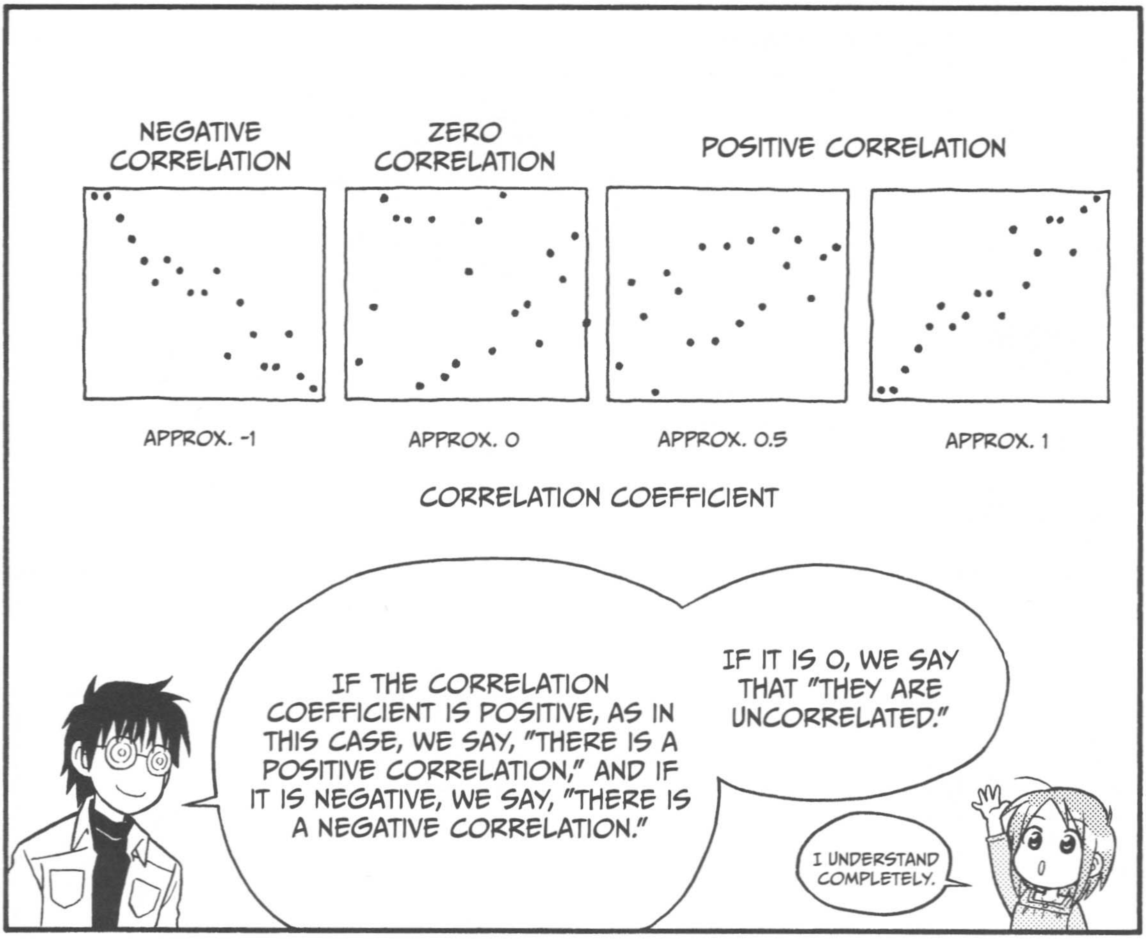
$$\frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \times \sum(y-\bar{y})^2}} = \frac{Sxy}{\sqrt{Sxx \times Syy}}$$

THE PROCESS FOR CALCULATING THE CORRELATION COEFFICIENT FOR MONTHLY EXPENDITURES ON MAKEUP AND CLOTHES

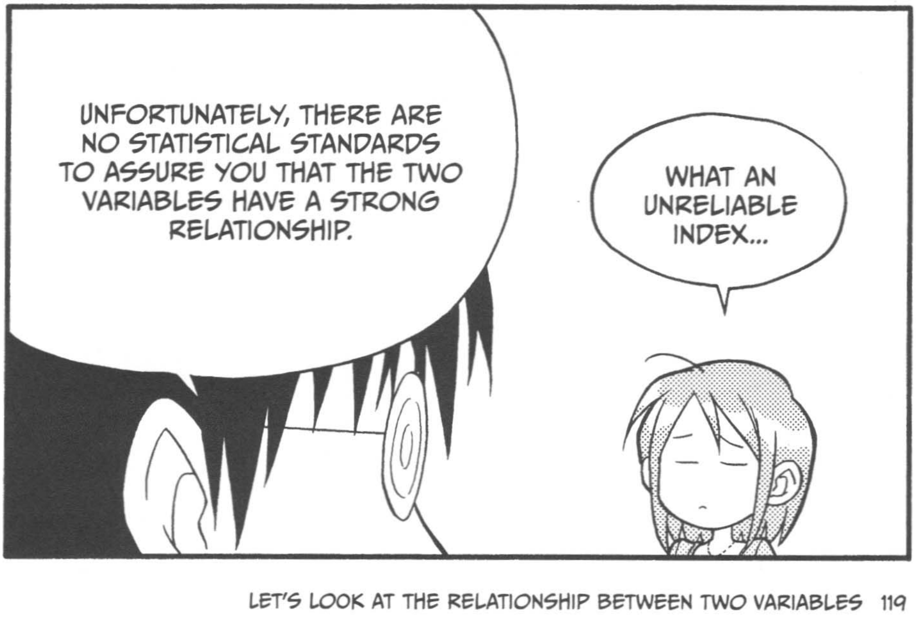
	Amount spent on makeup (¥)	Amount spent on clothes (¥)					
	x	y	$x-\bar{x}$	$y-\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	$(x-\bar{x})(y-\bar{y})$
Ms. A	3,000	7,000	-4,300	-8,000	18,490,000	64,000,000	34,400,000
Ms. B	5,000	8,000	-2,300	-7,000	5,290,000	49,000,000	16,100,000
Ms. C	12,000	25,000	4,700	10,000	22,090,000	100,000,000	47,000,000
Ms. D	2,000	5,000	-5,300	-10,000	28,090,000	100,000,000	53,000,000
Ms. E	7,000	12,000	-300	-3,000	90,000	9,000,000	900,000
Ms. F	15,000	30,000	7,700	15,000	59,290,000	225,000,000	115,500,000
Ms. G	5,000	10,000	-2,300	-5,000	5,290,000	25,000,000	11,500,000
Ms. H	6,000	15,000	-1,300	0	1,690,000	0	0
Ms. I	8,000	20,000	700	5,000	490,000	25,000,000	3,500,000
Ms. J	10,000	18,000	2,700	3,000	7,290,000	9,000,000	8,100,000
Sum	73,000	150,000	0	0	148,100,000	606,000,000	290,000,000
Mean	7,300	15,000					

$\bar{x}$ $\bar{y}$ Sxx Syy Sxy


NOW, ASSIGN THE VALUES TO THE FORMULA.
$\frac{Sxy}{\sqrt{Sxx \times Syy}} = \frac{290{,}000{,}000}{\sqrt{148{,}100{,}000 \times 606{,}000{,}000}} = 0.9680$
IT'S EASY IF YOU HAVE A CALCULATOR.
THE CORRELATION COEFFICIENT IS...0.9680!
THE CORRELATION COEFFICIENT GETS CLOSER TO ±1 IF THE LINEAR RELATIONSHIP BETWEEN THE TWO VARIABLES IS STRONGER.
AS THE RELATIONSHIP GETS WEAKER, IT GETS CLOSER TO 0.
-1 ← 0 → +1
THAT'S INTERESTING.
THE RESULT I JUST CALCULATED IS QUITE CLOSE TO 1, SO THAT IMPLIES THAT MAKEUP EXPENDITURES AND CLOTHING EXPENDITURES ARE VERY CLOSELY RELATED.
P-girls.
YOU ARE QUITE CORRECT.
WHEN DOES IT GET CLOSE TO -1?
THAT WOULD HAPPEN IF THE CLOTHING EXPENDITURES FELL AS THE MAKEUP EXPENDITURES ROSE.


NEGATIVE CORRELATION
ZERO CORRELATION
POSITIVE CORRELATION
APPROX. -1
APPROX. 0
APPROX. 0.5
APPROX. 1
CORRELATION COEFFICIENT
IF THE CORRELATION COEFFICIENT IS POSITIVE, AS IN THIS CASE, WE SAY, "THERE IS A POSITIVE CORRELATION," AND IF IT IS NEGATIVE, WE SAY, "THERE IS A NEGATIVE CORRELATION."
IF IT IS 0, WE SAY THAT "THEY ARE UNCORRELATED."
I UNDERSTAND COMPLETELY.


NOW, ABOUT THE CORRELATION COEFFICIENT...


UNFORTUNATELY, THERE ARE NO STATISTICAL STANDARDS TO ASSURE YOU THAT THE TWO VARIABLES HAVE A STRONG RELATIONSHIP.
WHAT AN UNRELIABLE INDEX...

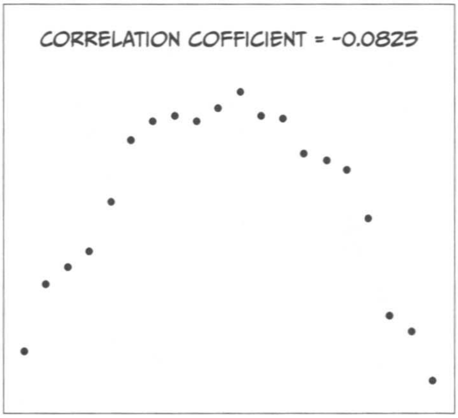
INFORMAL STANDARDS OF THE CORRELATION COEFFICIENT

Absolute value of the correlation coefficient		Detailed description	Rough description
1.0–0.9	⇨	Very strongly related	Related
0.9–0.7	⇨	Fairly strongly related	
0.7–0.5	⇨	Fairly weakly related	
Below 0.5	⇨	Very weakly related	Not related

WARNING

I mentioned earlier that the correlation coefficient is an index that shows the degree of *linear* relation between two numerical variables.

SAMPLE OF DATA UNSUITABLE FOR CORRELATION COEFFICIENT



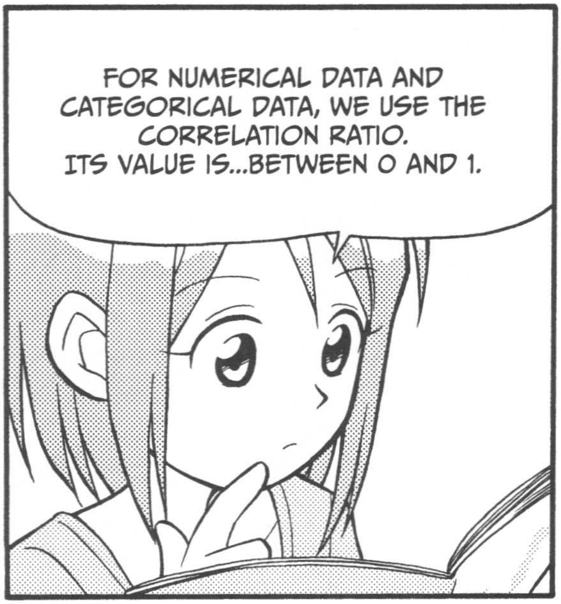
For example, the two variables are obviously related in this chart. However, the correlation coefficient is almost 0 because the relationship is *non-linear*.

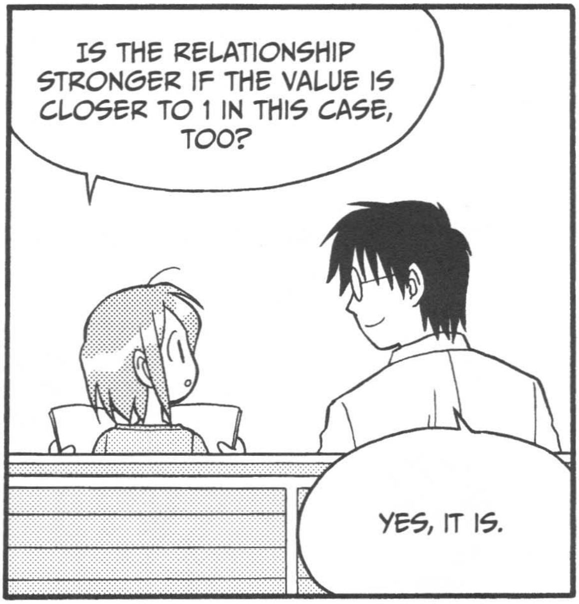
2. CORRELATION RATIO

Street Survey in Everyhills

Age and Favorite Fashion Brand

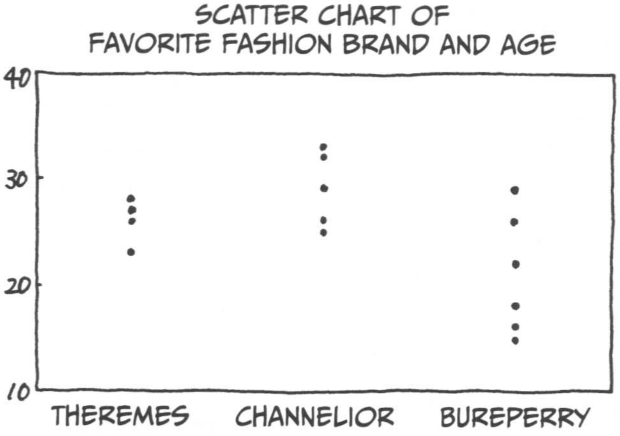
Respondent	Age	Brand
Ms. A	27	Theremes
Ms. B	33	Channelior
Ms. C	16	Bureperry
Ms. D	29	Bureperry
Ms. E	32	Channelior
Ms. F	23	Theremes
Ms. G	25	Channelior
Ms. H	28	Theremes
Ms. I	22	Bureperry
Ms. J	18	Bureperry
Ms. K	26	Channelior
Ms. L	26	Theremes
Ms. M	15	Bureperry
Ms. N	29	Channelior
Ms. O	26	Bureperry

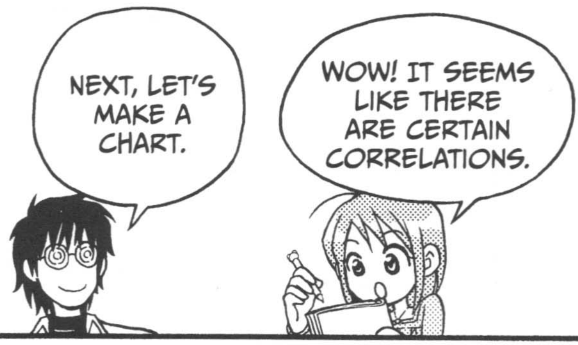


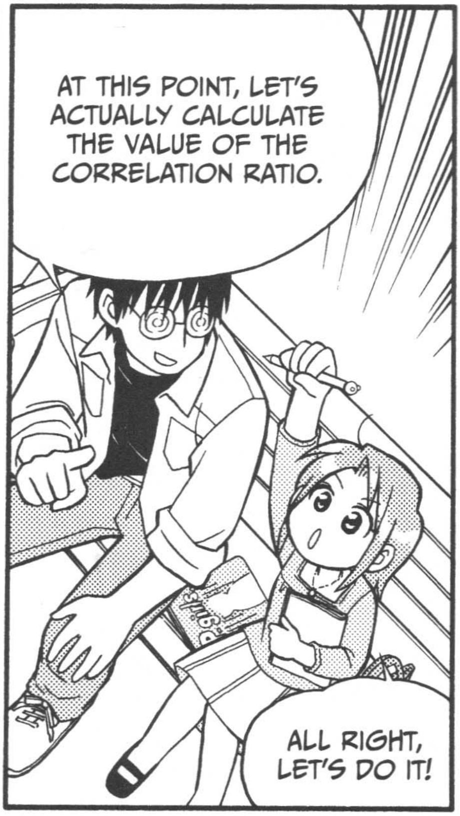


FAVORITE FASHION BRAND AND AGE

	THEREMES	CHANNELIOR	BUREPERRY	
	23	25	15	
	26	26	16	
	27	29	18	
	28	32	22	
		33	26	
			29	
SUM	104	145	126	375
MEAN	26	29	21	25







The value of the correlation ratio can be calculated by following steps 1 through 4 below.

Step 1

Do the calculations in the table below.

		Sum	
(Theremes – average for Theremes)²	$(23-26)^2 = (-3)^2 = 9$ $(26-26)^2 = 0^2 = 0$ $(27-26)^2 = 1^2 = 1$ $(28-26)^2 = 2^2 = 4$	14	S_{TT}
(Channelior – average for Channelior)²	$(25-29)^2 = (-4)^2 = 16$ $(26-29)^2 = (-3)^2 = 9$ $(29-29)^2 = 0^2 = 0$ $(32-29)^2 = 3^2 = 9$ $(33-29)^2 = 4^2 = 16$	50	S_{CC}
(Bureperry – average for Bureperry)²	$(15-21)^2 = (-6)^2 = 36$ $(16-21)^2 = (-5)^2 = 25$ $(18-21)^2 = (-3)^2 = 9$ $(22-21)^2 = 1^2 = 1$ $(26-21)^2 = 5^2 = 25$ $(29-21)^2 = 8^2 = 64$	160	S_{BB}

Step 2

Calculate the intraclass variance ($S_{TT} + S_{CC} + S_{BB}$ = how much the data within each category varies).

$$S_{TT} + S_{CC} + S_{BB} = 14 + 50 + 160 = 224$$

Step 3

Calculate the interclass variance, or how different the categories are from each other.

(number of votes for Theremes) × (average for Theremes − average for all data)^2
+ (number of votes for Channelior) × (average for Channelior − average for all data)^2
+ (number of votes for Bureperry) × (average for Bureperry − average for all data)^2

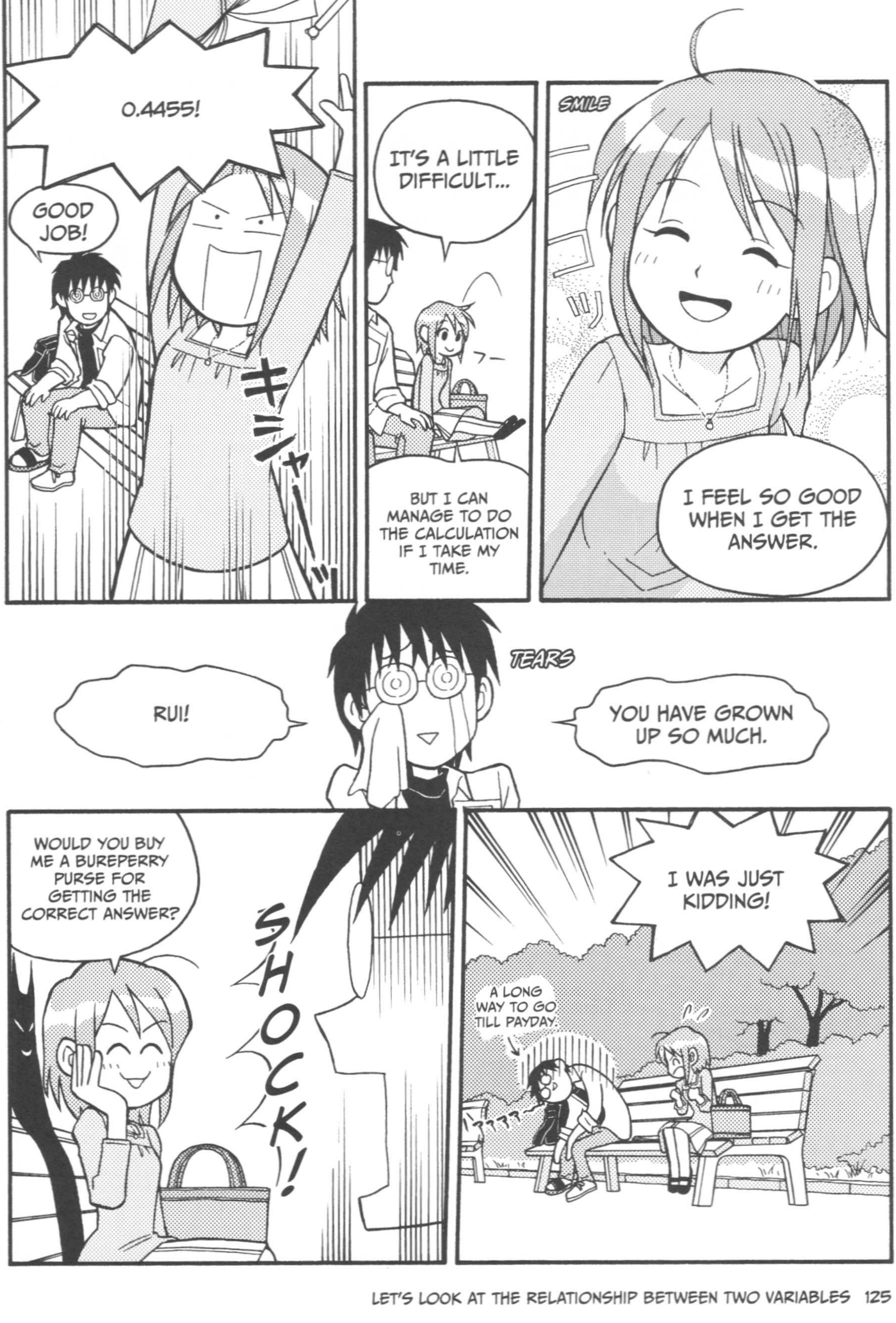
$$\begin{aligned} & 4 \times (26 - 25)^2 + 5 \times (29 - 25)^2 + 6 \times (21 - 25)^2 \\ &= 4 \times 1 + 5 \times 16 + 6 \times 16 \\ &= 4 + 80 + 96 \\ &= 180 \end{aligned}$$

Step 4

Calculate the value of the correlation ratio.

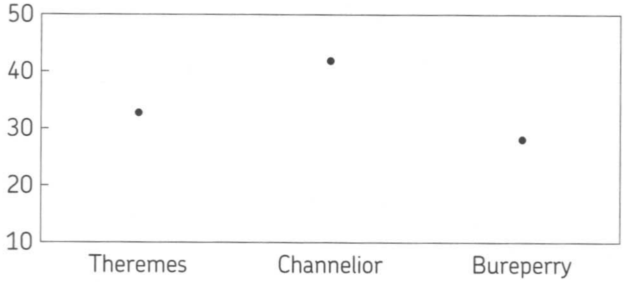
$$\frac{\text{interclass variance}}{\text{intraclass variance + interclass variance}}$$

$$\frac{180}{224 + 180} = \frac{180}{404} = 0.4455$$


0.4455!
GOOD JOB!
キシャーッ
IT'S A LITTLE DIFFICULT...
フー
BUT I CAN MANAGE TO DO THE CALCULATION IF I TAKE MY TIME.
SMILE
ニッ
I FEEL SO GOOD WHEN I GET THE ANSWER.
TEARS
RUI!
YOU HAVE GROWN UP SO MUCH.
WOULD YOU BUY ME A BUREPERRY PURSE FOR GETTING THE CORRECT ANSWER?
SHOCK!
I WAS JUST KIDDING!
A LONG WAY TO GO TILL PAYDAY.
ァァァァ

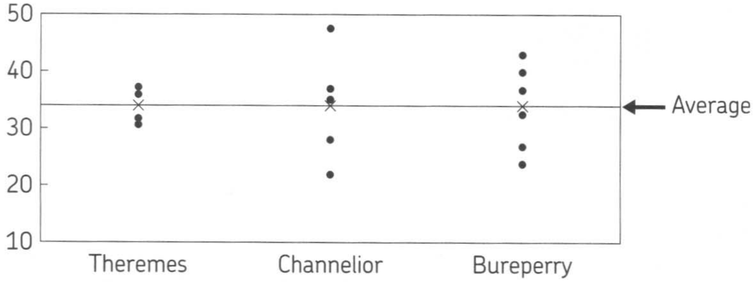
As explained earlier, the value of the correlation ratio is between 0 and 1. The stronger the correlation is between the two variables, the closer the value is to 1, and the weaker the correlation is between two variables, the closer the value is to 0. Refer to the charts below for more details.

Here is a scatter chart of favorite fashion brand and age (when the correlation ratio is 1).



correlation ratio is 1 ⇔ data included in each group is the same ⇔ intraclass variance is 0

Here is a scatter chart of favorite fashion brand and age (when the correlation ratio is 0).



correlation ratio is 0 ⇔ average for each group is the same ⇔ intraclass variance is 0

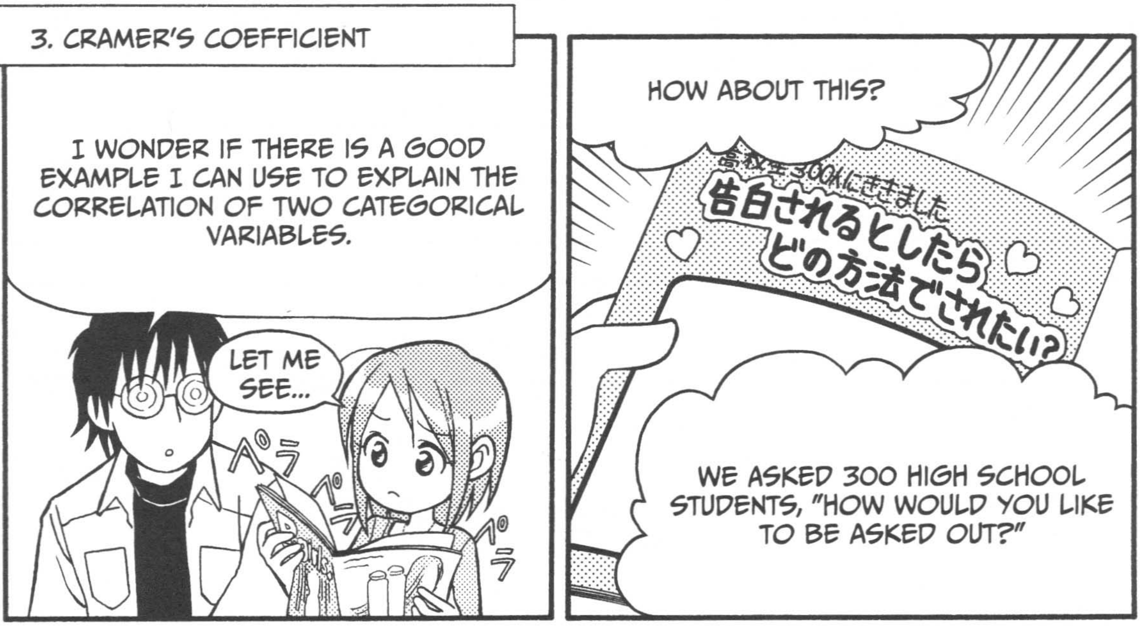
Unfortunately, there are no statistical standards such as "the two variables have a strong correlation if the correlation ratio is above a certain benchmark." However, informal standards are given below.

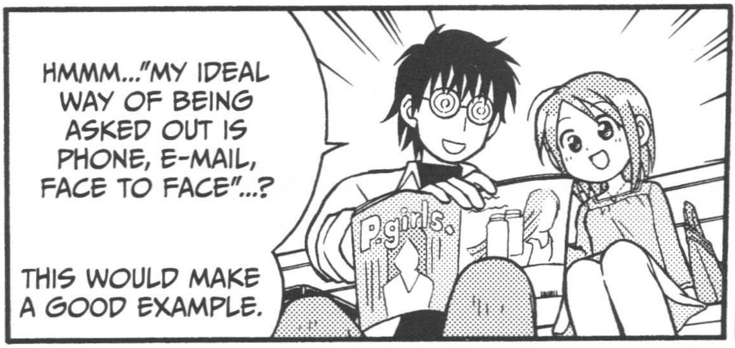
INFORMAL STANDARDS OF THE CORRELATION RATIO

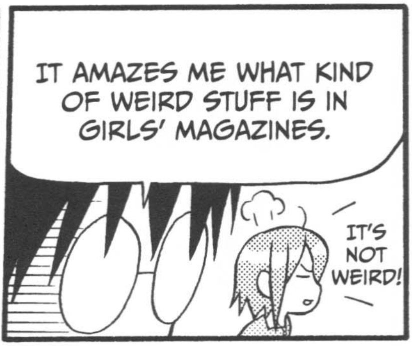
Correlation ratio		Detailed description	Rough description
1.0–0.8	⇨	Very strongly related	Related
0.8–0.5	⇨	Fairly strongly related	
0.5–0.25	⇨	Fairly weakly related	
Below 0.25	⇨	Very weakly related	Not related

The result of the calculation for the case in question was 0.4455, so the variables are fairly weakly related!

3. CRAMER'S COEFFICIENT







CROSS TABULATION OF SEX AND DESIRED WAY OF BEING ASKED OUT

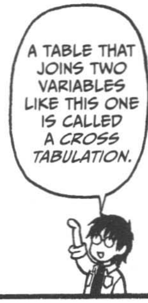
		DESIRED WAY OF BEING ASKED OUT			SUM
		PHONE	E-MAIL	FACE TO FACE	
SEX	FEMALE	34	61	53	148
	MALE	38	40	74	152
SUM		72	101	127	300

This indicates that 74 out of 152 males answered that they'd like to be asked out directly.

CROSS TABULATION OF SEX AND DESIRED WAY OF BEING ASKED OUT (HORIZONTAL PERCENTAGE TABLE)

		DESIRED WAY OF BEING ASKED OUT			SUM
		PHONE	E-MAIL	FACE TO FACE	
SEX	FEMALE	23%	41%	36%	100%
	MALE	25%	26%	49%	100%
SUM		24%	34%	42%	100%

This shows that 49% ($\frac{74}{152} \times 100$) of the 152 males would like to be asked out directly.




IT INDEED SEEMS THAT THERE IS A DIFFERENCE IN THE DESIRED WAY OF BEING ASKED OUT BETWEEN GIRLS AND BOYS.
IN OTHER WORDS, THERE IS A CORRELATION BETWEEN SEX AND DESIRED WAY OF BEING ASKED OUT.
I SHOULD CONTACT MR. IGARASHI DIRECTLY IF I AM GOING TO ASK HIM OUT...
RUI? ARE YOU LISTENING?
SURE, I AM.
WHAT WAS THE INDEX TO EXPRESS THE DEGREE OF CORRELATION BETWEEN TWO PIECES OF CATEGORICAL DATA?
THE CRAMER'S COEFFICIENT!
THE CRAMER'S COEFFICIENT IS ALSO CALLED THE *CRAMER'S V* OR AN *INDEPENDENT COEFFICIENT*.
I CANNOT CRAM SO MUCH NEW VOCABULARY INTO MY HEAD!
後悔*
WELL, THAT WAS KIND OF FUNNY.
YOU DON'T HAVE TO BE NICE TO ME.
* BAD JOKE AGAIN...

The Cramer's coefficient can be calculated by following steps 1 through 5 below.

Step 1

Prepare a cross tabulation. The values surrounded by the bold frame are called *actual measurement frequencies*.

		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	34	61	53	148
	Male	38	40	74	152
Sum		72	101	127	300

Step 2

Do the calculations in the table below. The values surrounded by the bold frame are called *expected frequencies*.

		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	$\frac{148 \times 72}{300}$	$\frac{148 \times 101}{300}$	$\frac{148 \times 127}{300}$	148
	Male	$\frac{152 \times 72}{300}$	$\frac{152 \times 101}{300}$	$\frac{152 \times 127}{300}$	152
Sum		72	101	127	300

$$\frac{\text{sum of male} \times \text{sum of face to face}}{\text{total number of values}}$$

Formula A

If sex and desired way of being asked out have no relationship, the ratio between phone, e-mail, and face to face should be

$$72 : 101 : 127 = \frac{72}{72 + 101 + 127} : \frac{101}{72 + 101 + 127} : \frac{127}{72 + 101 + 127}$$

$$= \frac{72}{300} : \frac{101}{300} : \frac{127}{300}$$

for both males and females, according to the sum column in the table in step 2. Thus, our expected frequency (Formula A) shows the predicted number of males who wish to be asked out directly when there is no relationship between sex and desired way of being asked out is 152 × (127 ÷ 300) = (152 × 127) ÷ 300, or

$$152 \times \frac{127}{300} = \frac{152 \times 127}{300} = 64.3$$

Step 3

Calculate $\dfrac{(\text{actual frequency} - \text{expected frequency})^2}{\text{expected frequency}}$ for each square.

		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	$\dfrac{\left(34 - \dfrac{148 \times 72}{300}\right)^2}{\dfrac{148 \times 72}{300}}$	$\dfrac{\left(61 - \dfrac{148 \times 101}{300}\right)^2}{\dfrac{148 \times 101}{300}}$	$\dfrac{\left(53 - \dfrac{148 \times 127}{300}\right)^2}{\dfrac{148 \times 127}{300}}$	148
	Male	$\dfrac{\left(38 - \dfrac{152 \times 72}{300}\right)^2}{\dfrac{152 \times 72}{300}}$	$\dfrac{\left(40 - \dfrac{152 \times 101}{300}\right)^2}{\dfrac{152 \times 101}{300}}$	$\dfrac{\left(74 - \dfrac{152 \times 127}{300}\right)^2}{\dfrac{152 \times 127}{300}}$	152
Sum		72	101	127	300

The bigger the gap between the actual frequencies and the expected frequencies, the larger the values in each square become.

Step 4

Calculate the sum of the value inside the bold frame in the table of step 3. This value is called *Pearson's chi-square test statistic*. It will be written as χ_0^2 from now on.

$$\chi_0^2 = \frac{\left(34 - \frac{148 \times 72}{300}\right)^2}{\frac{148 \times 72}{300}} + \frac{\left(61 - \frac{148 \times 101}{300}\right)^2}{\frac{148 \times 101}{300}} + \frac{\left(53 - \frac{148 \times 127}{300}\right)^2}{\frac{148 \times 127}{300}}$$

$$+ \frac{\left(38 - \frac{152 \times 72}{300}\right)^2}{\frac{152 \times 72}{300}} + \frac{\left(40 - \frac{152 \times 101}{300}\right)^2}{\frac{152 \times 101}{300}} + \frac{\left(74 - \frac{152 \times 127}{300}\right)^2}{\frac{152 \times 127}{300}}$$

$$= 8.0091$$

As can be understood from step 3, the more the actual measurements diverge from their expected frequencies, or the greater the correlation between sex and desired way of being asked out, the larger Pearson's chi-square test statistic (χ_0^2) becomes.

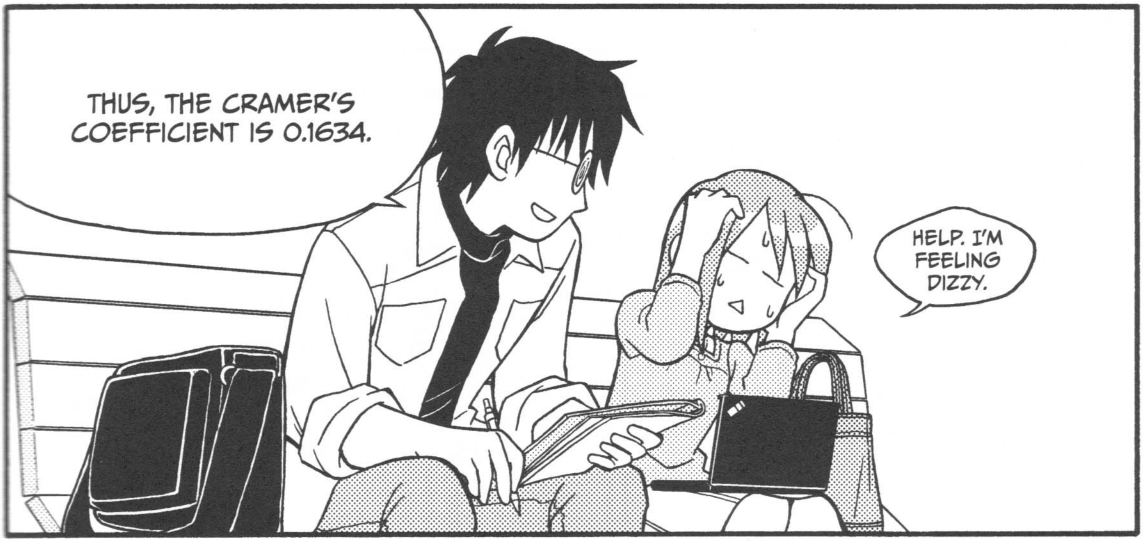
Step 5

Calculate the Cramer's coefficient.

$$\sqrt{\frac{\chi_0^{\,2}}{\text{the total number of values} \times \text{(min\{the number of lines in the cross tabulation, the number of rows in the cross tabulation\} – 1)}}}$$

min{*a*,*b*} means "whichever is smaller, *a* or *b*."

$$\sqrt{\frac{8.0091}{300 \times \min\{2,3\} - 1}} = \sqrt{\frac{8.0091}{300 \times (2 - 1)}} = \sqrt{\frac{8.0091}{300}} = 0.1634$$




I DON'T THINK I CAN DO THIS CALCULATION ALONE.
YOU'LL BE ALL RIGHT! THE CALCULATION IS A BIT COMPLICATED, BUT I AM SURE YOU CAN DO IT IF YOU FOLLOW THE STEPS.
IF THERE IS ANYTHING YOU CAN'T UNDERSTAND, JUST ASK ME, OK?
!
STARE...
WHY TURN AWAY?
OH!
WHAT'S WRONG?
NOTHING.
RUB RUB
FOR A MOMENT, MR. YAMAMOTO SOMEHOW SEEMED KIND OF GOOD LOOKING.

As explained earlier, the Cramer's coefficient is between 0 and 1. The stronger the correlation between two variables, the closer the coefficient gets to 1, and the weaker the correlation, the closer the coefficient gets to 0. See the cross tabulation (horizontal percentage table) below for more details.

Here is the cross tabulation of sex and desired way of being asked out (horizontal percentage table) when the value of the Cramer's coefficient is 1.

		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	17%	83%	0%	100%
	Male	0%	0%	100%	100%

Cramer's coefficient is 1 ⇔ the preferences of female and male are completely different

Here is the cross tabulation of sex and desired way of being asked out (horizontal percentage table) when the value of the Cramer's coefficient is 0.

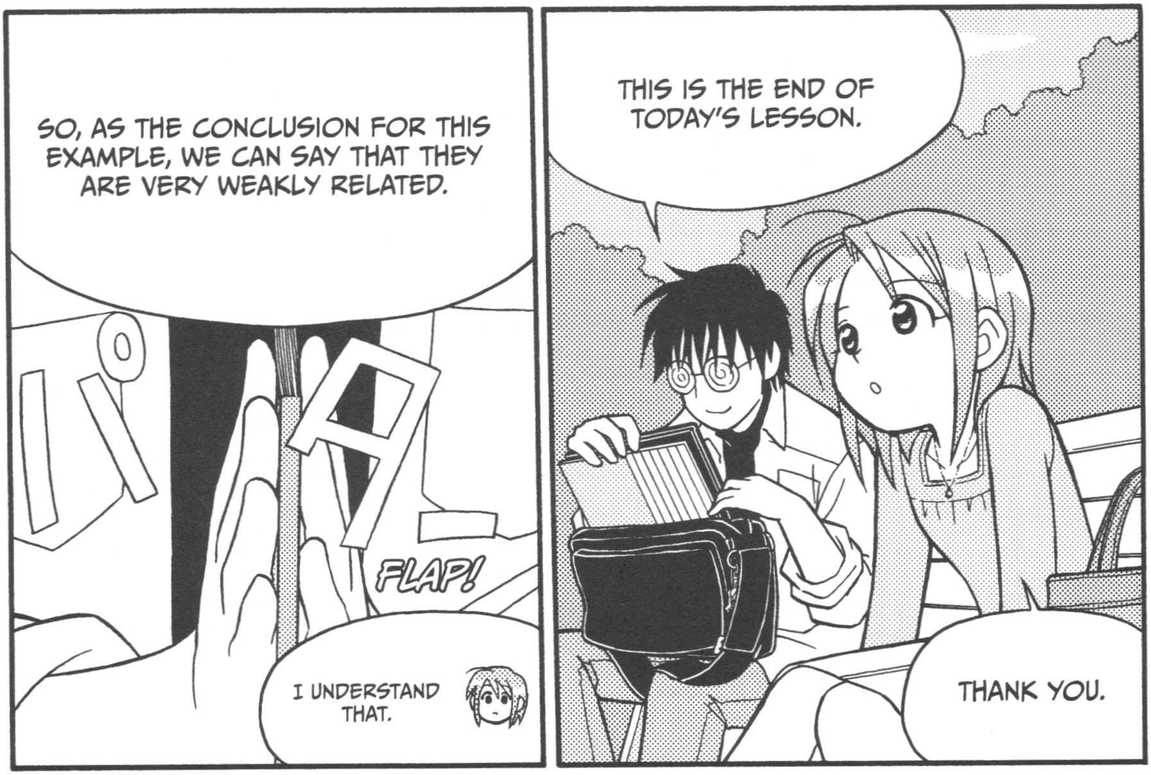
		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	17%	48%	35%	100%
	Male	17%	48%	35%	100%

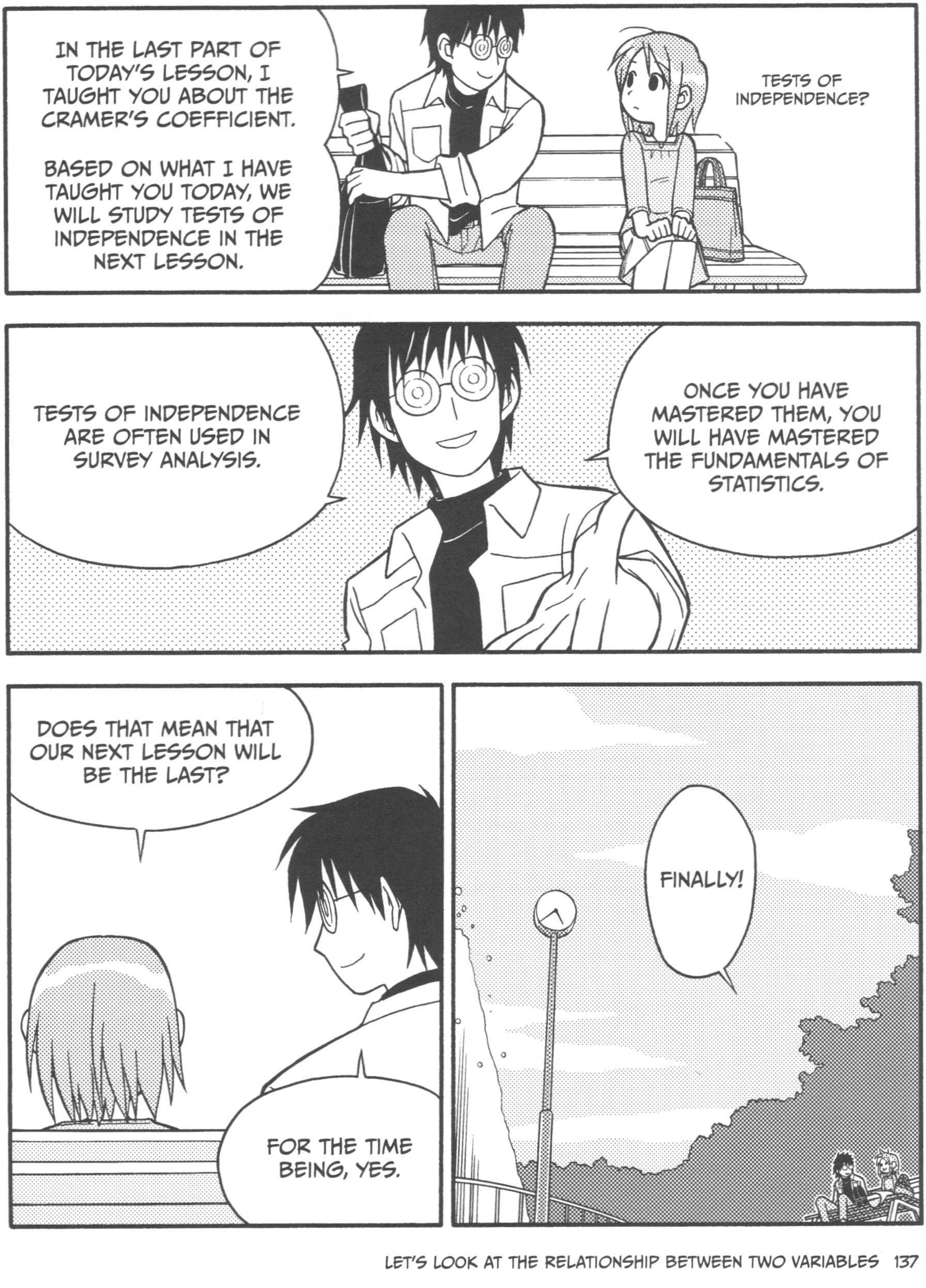
Cramer's coefficient is 0 ⇔ the preferences of female and male are the same

Unfortunately, there are no statistical standards such as "the two variables have a strong correlation if the Cramer's coefficient is above a certain benchmark." However, informal standards are given below.

INFORMAL STANDARDS OF THE CRAMER'S COEFFICIENT

Cramer's coefficient		Detailed description	Rough description
1.0–0.8	⇨	Very strongly related	Related
0.8–0.5	⇨	Fairly strongly related	
0.5–0.25	⇨	Fairly weakly related	
Below 0.25	⇨	Very weakly related	Not related




IN THE LAST PART OF TODAY'S LESSON, I TAUGHT YOU ABOUT THE CRAMER'S COEFFICIENT.
BASED ON WHAT I HAVE TAUGHT YOU TODAY, WE WILL STUDY TESTS OF INDEPENDENCE IN THE NEXT LESSON.
TESTS OF INDEPENDENCE?
TESTS OF INDEPENDENCE ARE OFTEN USED IN SURVEY ANALYSIS.
ONCE YOU HAVE MASTERED THEM, YOU WILL HAVE MASTERED THE FUNDAMENTALS OF STATISTICS.
DOES THAT MEAN THAT OUR NEXT LESSON WILL BE THE LAST?
FOR THE TIME BEING, YES.
FINALLY!

EXERCISE AND ANSWER

EXERCISE

Company *X* runs a casual dining restaurant. Its financial status was declining recently. Thus, Company *X* decided to study its customers' needs and conducted a survey of randomly chosen people, age 20 or older, residing in Japan. The table below shows the results of this survey.

Respondent	**What food do you often have in a casual dining restaurant?**	**If a free drink is to be served after a meal, which would you prefer? Coffee or tea?**
1	Chinese	Coffee
2	European	Coffee
...	...	...
250	Japanese	Tea

Below is a cross tabulation made using the table above.

		Preference for coffee or tea		Sum
		Coffee	Tea	
Type of food often ordered	Japanese	43	33	76
	European	51	53	104
	Chinese	29	41	70
Sum		123	127	250

Calculate the Cramer's coefficient for the food often ordered in casual dining restaurants and the preferred free drink of either coffee or tea.

ANSWER

Step 1

Prepare a cross tabulation.

		Preference for coffee or tea		Sum
		Coffee	Tea	
Type of food often ordered	Japanese	43	33	76
	European	51	53	104
	Chinese	29	41	70
Sum		123	127	250

Step 2

Calculate the expected frequency.

		Preference for coffee or tea		Sum
		Coffee	Tea	
Type of food often ordered	Japanese	$\frac{76 \times 123}{250}$	$\frac{76 \times 127}{250}$	76
	European	$\frac{104 \times 123}{250}$	$\frac{104 \times 127}{250}$	104
	Chinese	$\frac{70 \times 123}{250}$	$\frac{70 \times 127}{250}$	70
Sum		123	127	250

Step 3

Calculate

$$\frac{(\text{actual measurement frequency} - \text{expected frequency})^2}{\text{expected frequency}}$$

for each square.

		Preference for coffee or tea		Sum
		Coffee	Tea	
Type of food often ordered	Japanese	$\dfrac{\left(43 - \dfrac{76 \times 123}{250}\right)^2}{\dfrac{76 \times 123}{250}}$	$\dfrac{\left(33 - \dfrac{76 \times 127}{250}\right)^2}{\dfrac{76 \times 127}{250}}$	76
	European	$\dfrac{\left(51 - \dfrac{104 \times 123}{250}\right)^2}{\dfrac{104 \times 123}{250}}$	$\dfrac{\left(53 - \dfrac{104 \times 127}{250}\right)^2}{\dfrac{104 \times 127}{250}}$	104
	Chinese	$\dfrac{\left(29 - \dfrac{70 \times 123}{250}\right)^2}{\dfrac{70 \times 123}{250}}$	$\dfrac{\left(41 - \dfrac{70 \times 127}{250}\right)^2}{\dfrac{70 \times 127}{250}}$	70
Sum		123	127	250

Step 4

Calculate the sum of the value inside the bold frame in the table in step 3, which is the value of Pearson's chi-square test statistic (χ_0^2).

$$\chi_0^2 = \frac{\left(43 - \frac{76 \times 123}{250}\right)^2}{\frac{76 \times 123}{250}} + \frac{\left(33 - \frac{76 \times 127}{250}\right)^2}{\frac{76 \times 127}{250}}$$

$$+ \frac{\left(51 - \frac{104 \times 123}{250}\right)^2}{\frac{104 \times 123}{250}} + \frac{\left(53 - \frac{104 \times 127}{250}\right)^2}{\frac{104 \times 127}{250}}$$

$$+ \frac{\left(29 - \frac{70 \times 123}{250}\right)^2}{\frac{70 \times 123}{250}} + \frac{\left(41 - \frac{70 \times 127}{250}\right)^2}{\frac{70 \times 127}{250}}$$

$$= 3.3483$$

Step 5

Calculate the Cramer's coefficient.

$$\sqrt{\frac{\chi_0^2}{\text{the total number of values} \times \left(\min\left\{\text{the number of lines in the cross tabulation}, \text{the number of rows in the cross tabulation}\right\} - 1\right)}}$$

$$\sqrt{\frac{3.3483}{250 \times (\min\{3,2\} - 1)}} = \sqrt{\frac{3.3483}{250 \times (2 - 1)}} = \sqrt{\frac{3.3483}{250}} = 0.1157$$

SUMMARY

- The index used to describe the degree of correlation between numerical data and numerical data is the *correlation coefficient*.
- The index used to describe the degree of correlation between numerical data and categorical data is the *correlation ratio*.
- The index used to describe the degree of correlation between categorical data and categorical data is the *Cramer's coefficient* (sometimes called the *Cramer's V* or an *independent coefficient*).
- The characteristics of the correlation coefficient, correlation ratio, and Cramer's coefficient are shown in the table below.

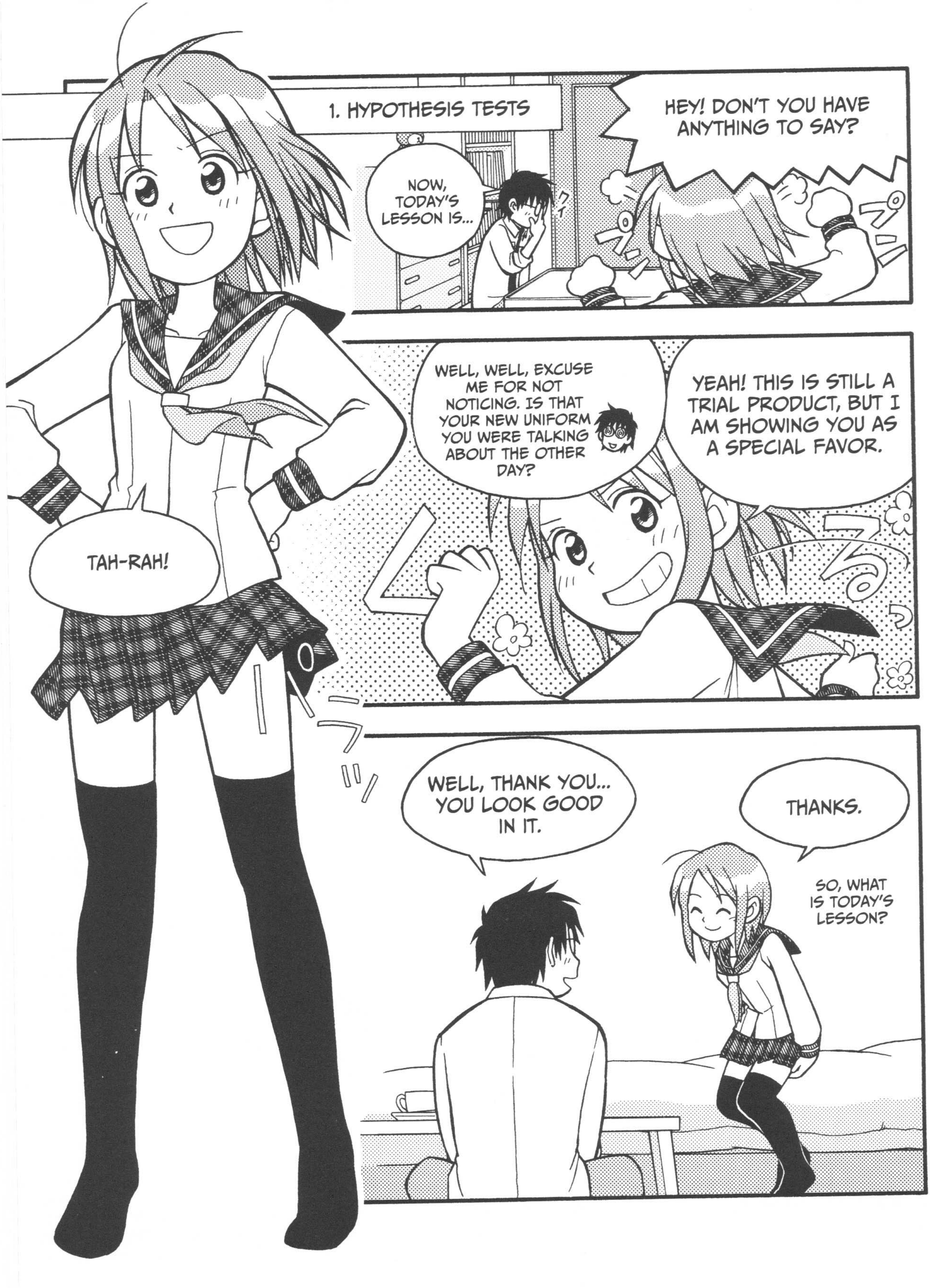
	Minimum	**Maximum**	**The value when the two variables are not correlated at all**	**The value when the two variables are most strongly correlated**
Correlation coefficient	–1	1	0	–1 or 1
Correlation ratio	0	1	0	1
Cramer's coefficient	0	1	0	1

- There are no statistical standards for the correlation coefficient, correlation ratio, and Cramer's coefficient, such as "the two variables have a strong correlation if the value is above a certain benchmark."

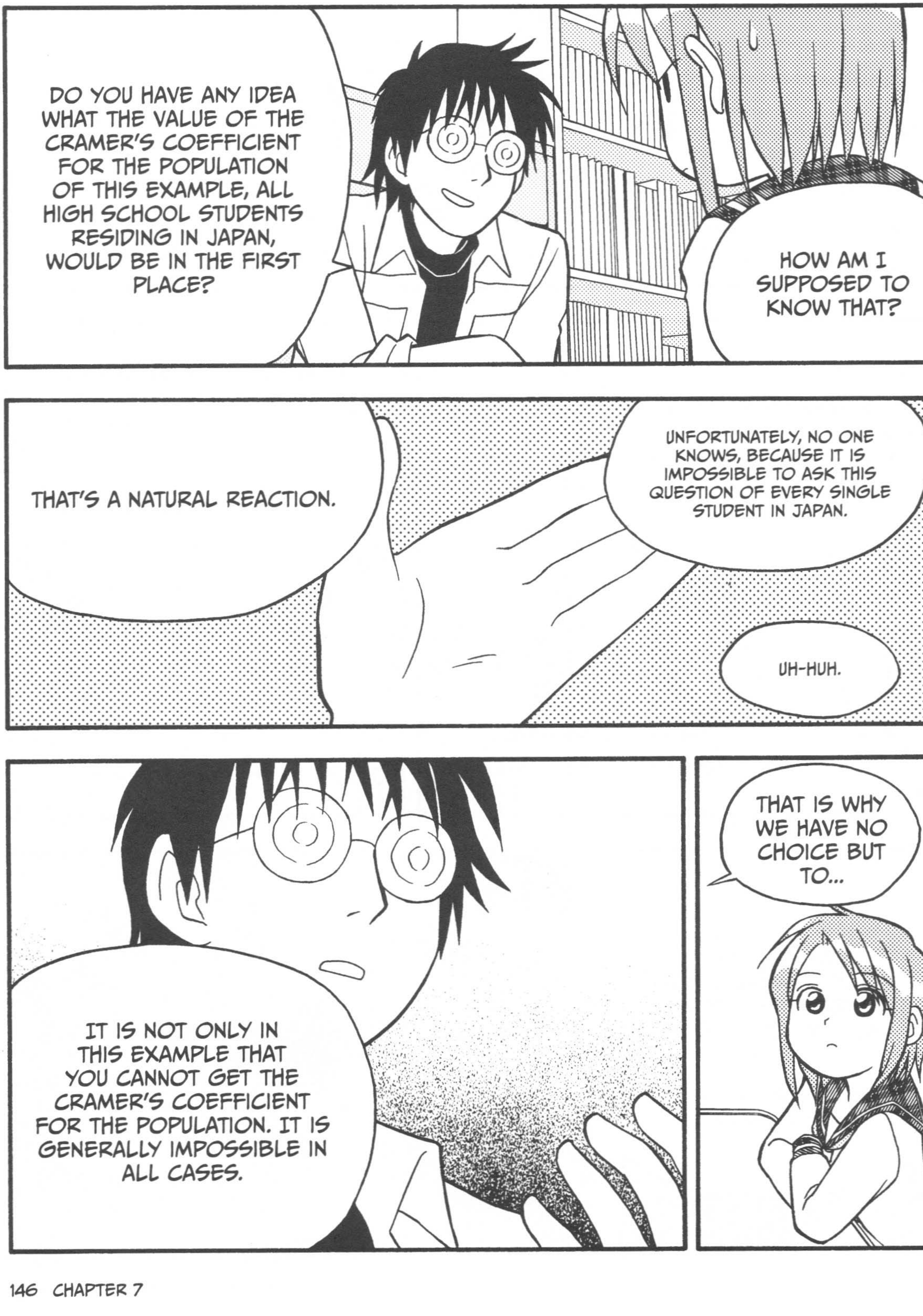
7

LET'S EXPLORE THE HYPOTHESIS TESTS

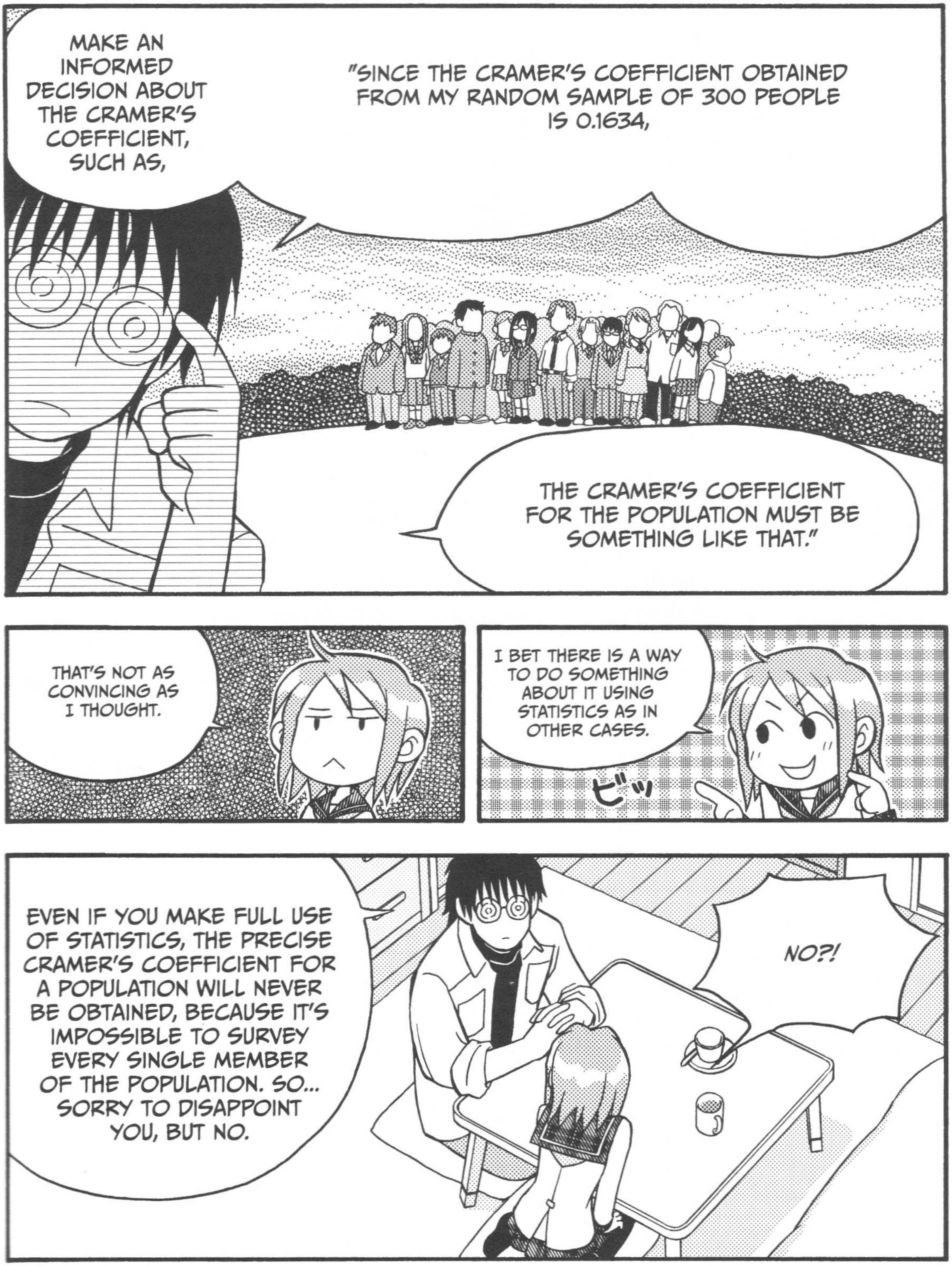
1. HYPOTHESIS TESTS



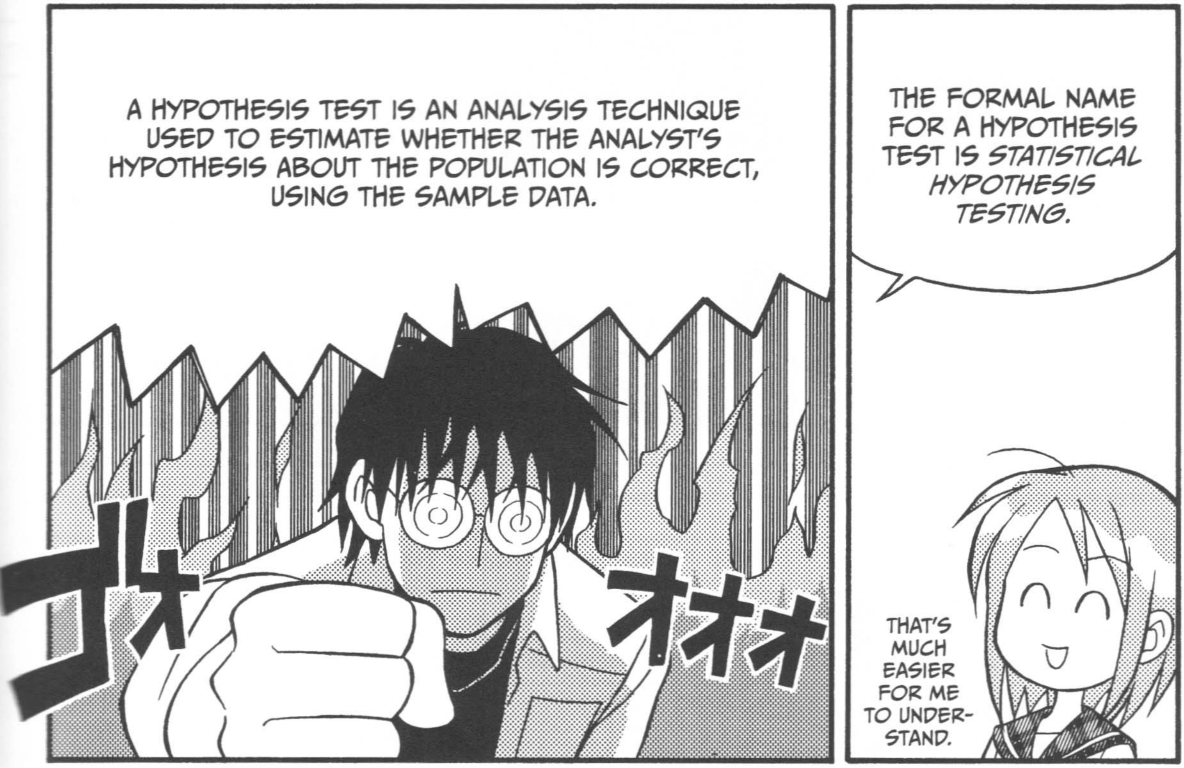

REMEMBER LEARNING ABOUT THE CRAMER'S COEFFICIENT IN OUR LAST LESSON?
高校生300人にききました
告白されるとしたらどの方法でされたい!?*
*WE ASKED 300 HIGH SCHOOL STUDENTS, "HOW WOULD YOU LIKE TO BE ASKED OUT?"
YOU MEAN THAT SURVEY ON HOW TO ASK OUT SOMEONE?
IN THAT EXAMPLE, THE CRAMER'S COEFFICIENT WAS 0.1634, AND THE RESULT TURNED OUT TO BE VERY WEAKLY CORRELATED.
YES, I REMEMBER THAT.
NOW, THINK CAREFULLY, RUI.
THE RESULT OF THAT SURVEY WAS OBTAINED FROM THE RESPONSES OF 300 PEOPLE...
...WHO WERE CHOSEN RANDOMLY FROM ALL HIGH SCHOOL STUDENTS RESIDING IN JAPAN.
IF A DIFFERENT 300 PEOPLE WERE CHOSEN, THE CRAMER'S COEFFICIENT WOULD NOT HAVE BEEN 0.1634.
COME TO THINK OF IT, YOU ARE RIGHT.



DO YOU HAVE ANY IDEA WHAT THE VALUE OF THE CRAMER'S COEFFICIENT FOR THE POPULATION OF THIS EXAMPLE, ALL HIGH SCHOOL STUDENTS RESIDING IN JAPAN, WOULD BE IN THE FIRST PLACE?
HOW AM I SUPPOSED TO KNOW THAT?
THAT'S A NATURAL REACTION.
UNFORTUNATELY, NO ONE KNOWS, BECAUSE IT IS IMPOSSIBLE TO ASK THIS QUESTION OF EVERY SINGLE STUDENT IN JAPAN.
UH-HUH.
IT IS NOT ONLY IN THIS EXAMPLE THAT YOU CANNOT GET THE CRAMER'S COEFFICIENT FOR THE POPULATION. IT IS GENERALLY IMPOSSIBLE IN ALL CASES.
THAT IS WHY WE HAVE NO CHOICE BUT TO...


MAKE AN INFORMED DECISION ABOUT THE CRAMER'S COEFFICIENT, SUCH AS,
"SINCE THE CRAMER'S COEFFICIENT OBTAINED FROM MY RANDOM SAMPLE OF 300 PEOPLE IS 0.1634,
THE CRAMER'S COEFFICIENT FOR THE POPULATION MUST BE SOMETHING LIKE THAT."
THAT'S NOT AS CONVINCING AS I THOUGHT.
I BET THERE IS A WAY TO DO SOMETHING ABOUT IT USING STATISTICS AS IN OTHER CASES.
ビッ
EVEN IF YOU MAKE FULL USE OF STATISTICS, THE PRECISE CRAMER'S COEFFICIENT FOR A POPULATION WILL NEVER BE OBTAINED, BECAUSE IT'S IMPOSSIBLE TO SURVEY EVERY SINGLE MEMBER OF THE POPULATION. SO... SORRY TO DISAPPOINT YOU, BUT NO.
NO?!


ぱあああ。。。
HOWEVER, THERE IS A WAY TO FIND OUT IF...
...THE CRAMER'S COEFFICIENT FOR A POPULATION IS NOT ZERO!
IT DOESN'T SEEM LIKE A BIG DEAL TO ME.
?
OF COURSE IT'S A BIG DEAL. IT GIVES YOU AN OBJECTIVE VIEW ABOUT THE POPULATION.
HOW DO YOU DO IT?
I HAVE MENTIONED IN OUR PREVIOUS LESSON THAT WE WILL STUDY TESTS OF INDEPENDENCE TODAY. THIS IS THE MEANS OF ANALYSIS WE ARE GOING TO USE NOW.
IS THAT SOMETHING LIKE THE TEST OF ENGLISH PROFICIENCY?
NO, NO, IT'S COMPLETELY DIFFERENT.
A TEST OF INDEPENDENCE IS A KIND OF ANALYSIS TECHNIQUE THAT IS GENERALLY REFERRED TO AS A HYPOTHESIS TEST IN STATISTICS.
TESTS OF INDEPENDENCE
HYPOTHESIS TESTS
TESTS OF CORRELATION RATIO
TESTS OF DIFFERENCE BETWEEN POPULATION MEANS
TESTS OF DIFFERENCE BETWEEN POPULATION RATIOS
LET ME EXPLAIN HYPOTHESIS TESTS FIRST.
TESTS OF CORRELATION
YES, PLEASE.



EXAMPLES OF HYPOTHESIS TESTS

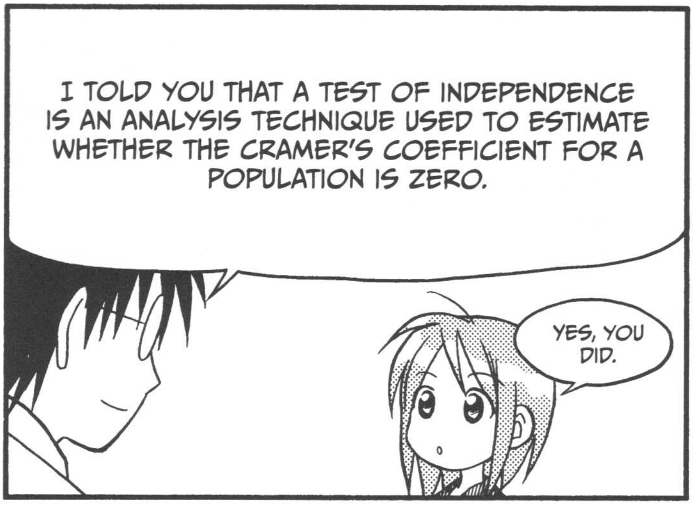
Name	**Example of use**
Tests of independence	Estimates whether the value of the Cramer's coefficient for sex and desired way of being asked out is zero for a population
Tests of correlation ratio	Estimates whether the value of the correlation ratio for favorite fashion brand and age is zero for a population
Tests of correlation	Estimates whether the correlation coefficient for amount spent on makeup and amount spent on clothes is zero for a population
Tests of difference between population means	Estimates whether allowances are different between high school girls in Tokyo and Osaka*
Tests of difference between population ratios	Estimates whether the approval rating of cabinet *X* is different between voters residing in urban areas and rural areas*

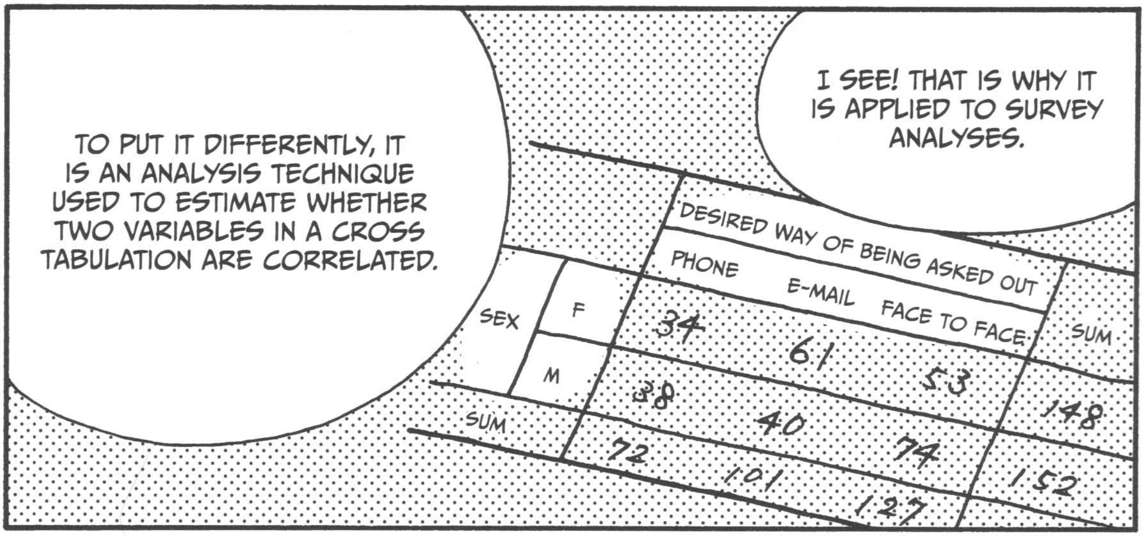
* Note that two populations are being considered.

PROCEDURE FOR A HYPOTHESIS TEST

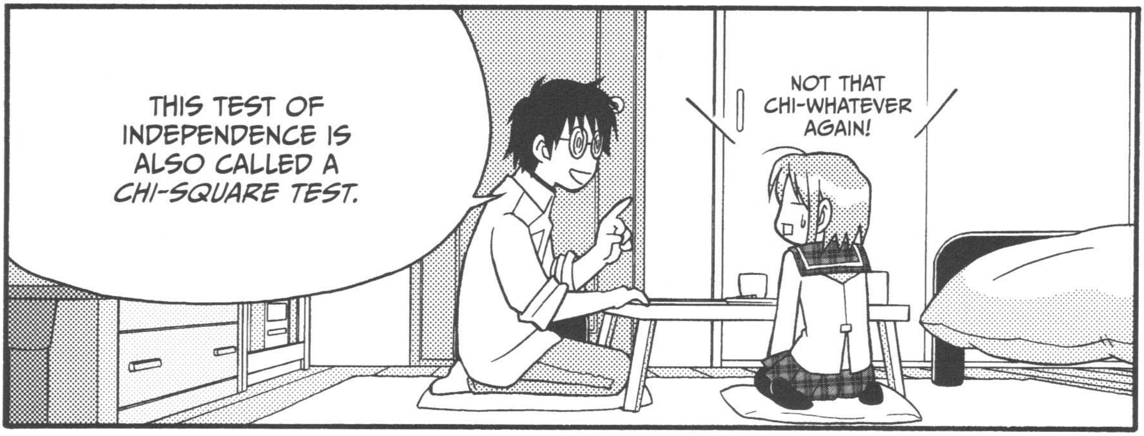
Step 1	Define the population.
Step 2	Set up a null hypothesis and an alternative hypothesis.
Step 3	Select which hypothesis test to conduct.
Step 4	Determine the significance level.
Step 5	Obtain the test statistic from the sample data.
Step 6	Determine whether the test statistic obtained in step 5 is in the critical region.
Step 7	If the test statistic is in the critical region, you must reject the null hypothesis. If not, you fail to reject the null hypothesis.

2. THE CHI-SQUARE TEST OF INDEPENDENCE





		DESIRED WAY OF BEING ASKED OUT			
		PHONE	E-MAIL	FACE TO FACE	SUM
SEX	F	34	61	53	148
	M	38	40	74	152
SUM		72	101	127	



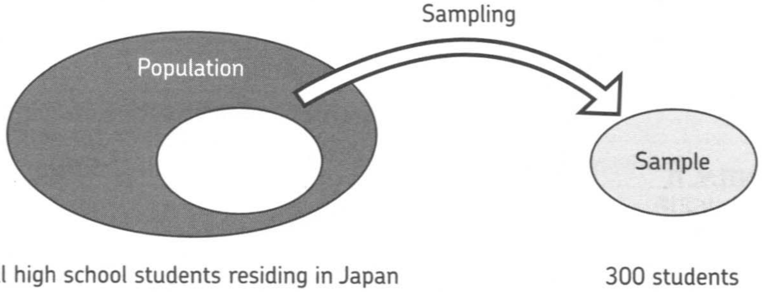
EXPLANATION

Pearson's chi-square test statistic (χ_0^2) and chi-square distribution

Before giving an actual example of a test of independence, I would like to explain an important fact that is fundamental to tests of independence. Though it is impossible to do this in reality, suppose the below experiment is conducted.

Step 1

Take a random sample of 300 students from the population "all high school students residing in Japan."



Step 2

Conduct the survey on page 127 with the 300 people chosen in step 1 to obtain the chi-square statistic (χ_0^2).

Step 3

Put the 300 people back into the population.

Step 4

Repeat steps 1 through 3 over and over.

In this experiment, if the value of the Cramer's coefficient for the population "all high school students residing in Japan" is 0, the graph of Pearson's chi-square test statistic (χ_0^2) turns out to be a chi-square distribution with 2 degrees of freedom. In other words, if the value of the Cramer's coefficient for the population "all high school students residing in Japan" is 0, then Pearson's chi-square test statistic (χ_0^2) follows a chi-square distribution with 2 degrees of freedom.

- See pages 130–133 for information on how to obtain Pearson's chi-square test statistic (χ_0^2).
- See page 100 for information on a chi-square distribution with 2 degrees of freedom.

We have actually conducted this experiment. In carrying out the experiment, we set the restrictions below.

- As it is impossible to experiment with the actual population of "all high school students residing in Japan," the group of 10,000 people in Table 7-1 will be regarded as "all high school students residing in Japan" instead.
- We assume that the Cramer's coefficient for "all high school students residing in Japan" is 0. This means that the ratio of those who prefer being asked out by phone to those who prefer being asked out by e-mail to those who prefer being asked out directly is equal for girls and boys (see page 135). The cross tabulation for Table 7-1 is Table 7-2.
- Since it is otherwise endless, we will stop repeating steps 1 through 3 after 10,000 times.

TABLE 7-1: DESIRED WAY OF BEING ASKED OUT (ALL HIGH SCHOOL STUDENTS RESIDING IN JAPAN)

Respondent	**Sex**	**Desired way of being asked out**
1	Female	Face to face
2	Female	Phone
...	...	...
10,000	Male	E-mail

TABLE 7-2: CROSS TABULATION OF SEX AND DESIRED WAY OF BEING ASKED OUT

		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	400	1,600	2,000	4,000
	Male	600	2,400	3,000	6,000
Sum		1,000	4,000	5,000	10,000

Table 7-3 shows the result of the experiment. Figure 7-1 is a histogram made according to Table 7-3.

TABLE 7-3: RESULT OF EXPERIMENT

Experiment	Pearson's chi-square test statistic (χ_0^2)
1	0.8598
2	0.7557
...	...
10,000	2.7953

FIGURE 7-1: A HISTOGRAM BASED ON TABLE 7-3 (RANGE OF CLASS = 1)

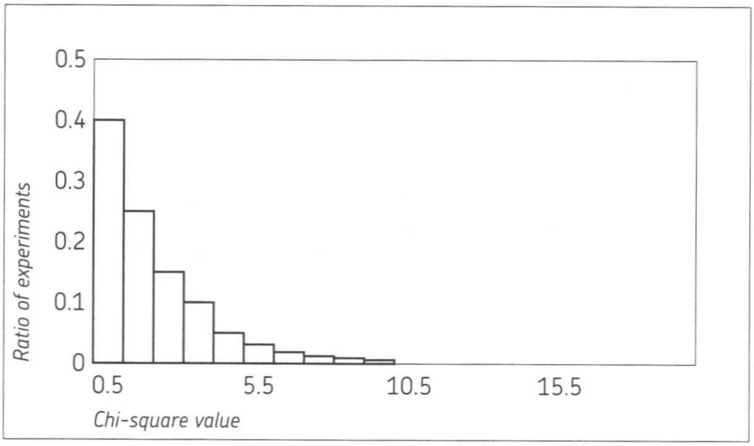
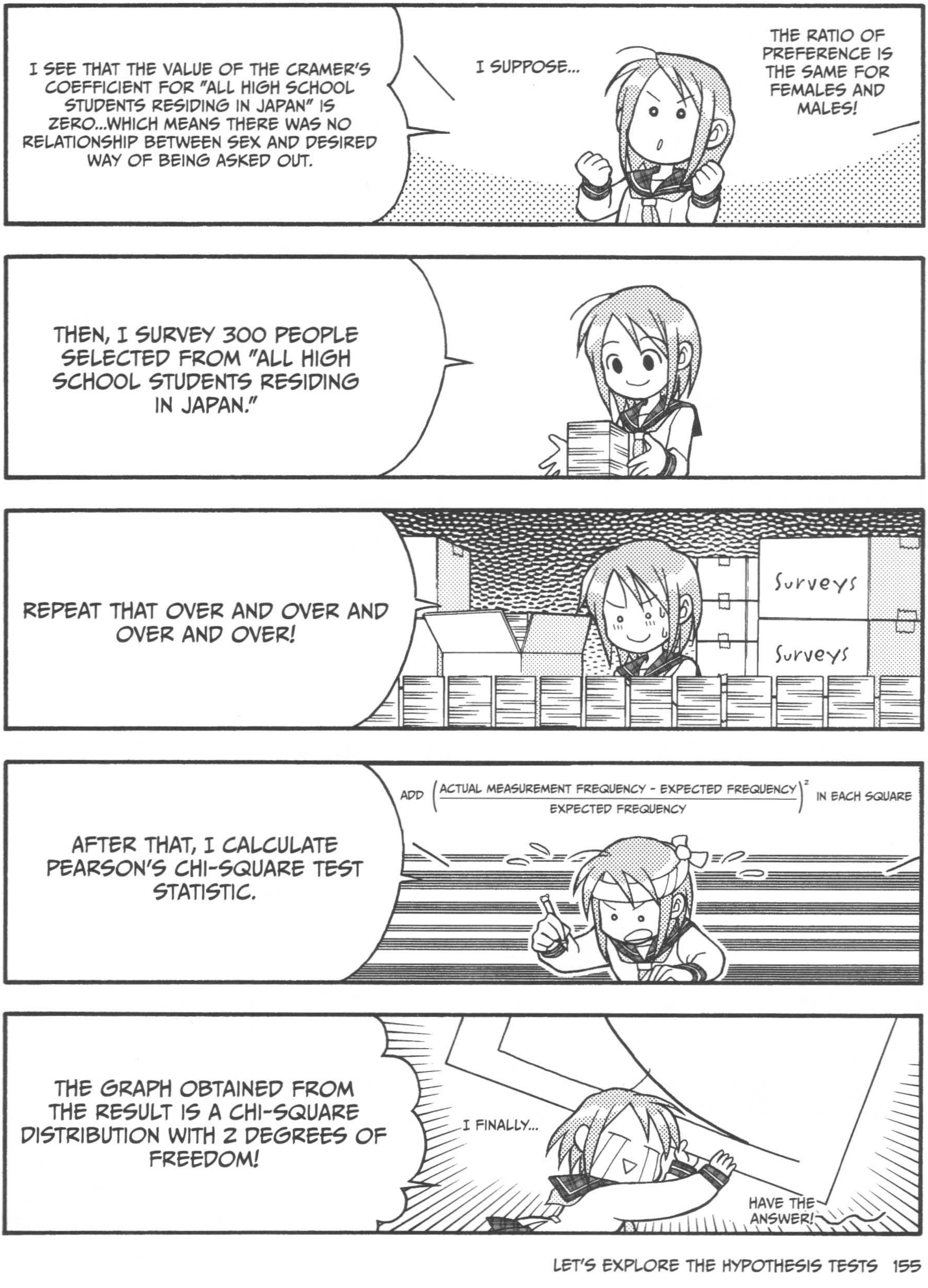
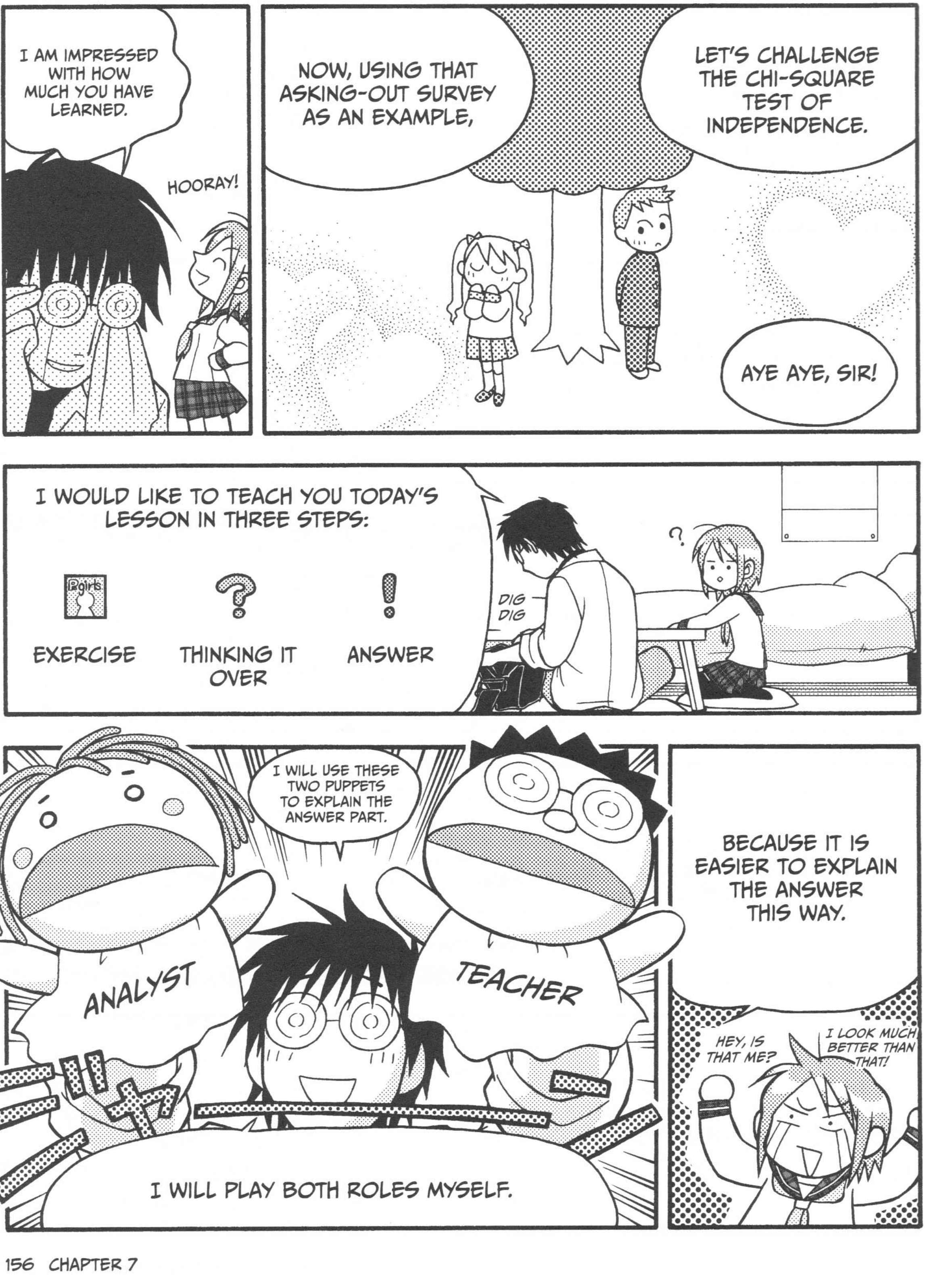


Figure 7-1 indeed looks similar to the graph on page 100, "2 Degrees of Freedom." It seems to be correct that Pearson's chi-square test statistic (χ_0^2) follows a chi-square distribution with 2 degrees of freedom. Though this has nothing to do with the experiment itself, here is one point to note. Two degrees of freedom comes from:

$$\underset{\substack{\uparrow \\ \text{2 patterns:} \\ \text{female and male}}}{(2 - 1)} \times \underset{\substack{\uparrow \\ \text{3 patterns:} \\ \text{phone, e-mail, and face to face}}}{(3 - 1)} = 1 \times 2 = 2$$

I will not go into why such a strange calculation is applied, as it is a topic much too advanced for the level of this book. But don't worry—even if you don't fully understand the calculation, you won't be at any disadvantage.


I SEE THAT THE VALUE OF THE CRAMER'S COEFFICIENT FOR "ALL HIGH SCHOOL STUDENTS RESIDING IN JAPAN" IS ZERO...WHICH MEANS THERE WAS NO RELATIONSHIP BETWEEN SEX AND DESIRED WAY OF BEING ASKED OUT.
I SUPPOSE...
THE RATIO OF PREFERENCE IS THE SAME FOR FEMALES AND MALES!
THEN, I SURVEY 300 PEOPLE SELECTED FROM "ALL HIGH SCHOOL STUDENTS RESIDING IN JAPAN."
REPEAT THAT OVER AND OVER AND OVER AND OVER!
Surveys
Surveys
ADD $\left(\frac{\text{ACTUAL MEASUREMENT FREQUENCY} - \text{EXPECTED FREQUENCY}}{\text{EXPECTED FREQUENCY}}\right)^2$ IN EACH SQUARE
AFTER THAT, I CALCULATE PEARSON'S CHI-SQUARE TEST STATISTIC.
THE GRAPH OBTAINED FROM THE RESULT IS A CHI-SQUARE DISTRIBUTION WITH 2 DEGREES OF FREEDOM!
I FINALLY...
HAVE THE ANSWER!


I AM IMPRESSED WITH HOW MUCH YOU HAVE LEARNED.
HOORAY!
NOW, USING THAT ASKING-OUT SURVEY AS AN EXAMPLE,
LET'S CHALLENGE THE CHI-SQUARE TEST OF INDEPENDENCE.
AYE AYE, SIR!
I WOULD LIKE TO TEACH YOU TODAY'S LESSON IN THREE STEPS:
Pgirls
EXERCISE
THINKING IT OVER
ANSWER
DIG DIG
?
I WILL USE THESE TWO PUPPETS TO EXPLAIN THE ANSWER PART.
ANALYST
TEACHER
I WILL PLAY BOTH ROLES MYSELF.
BECAUSE IT IS EASIER TO EXPLAIN THE ANSWER THIS WAY.
HEY, IS THAT ME?
I LOOK MUCH BETTER THAN THAT!

EXERCISE

P-Girls Magazine decided to publish an article titled "We Asked 300 High School Students, 'How Would You Like to Be Asked Out?'" In order to prepare the article, a journalist randomly chose 300 people from all the high school students residing in Japan and took a survey. The table below is the result of this survey.

Respondent	**Desired way of being asked out**	**Age**	**Sex**
1	Face to face	17	Female
2	Phone	15	Female
...	...	...	...
300	E-mail	18	Male

The table below is the cross tabulation of sex and desired way of being asked out.

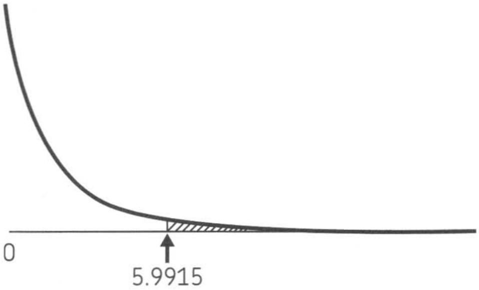
		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	34	61	53	148
	Male	38	40	74	152
Sum		72	101	127	300

Using the chi-square test of independence, estimate if the Cramer's coefficient for sex and desired way of being asked out in the population "all high school students residing in Japan" is greater than 0. This is the same as estimating with a test of independence whether sex and desired way of being asked out are correlated. Remember that the significance level (explained later) is 0.05.

THINKING IT OVER

As explained on pages 152–154, Pearson's chi-square test statistic (χ_0^2) follows a chi-square distribution with 2 degrees of freedom if the null hypothesis states that the value of the Cramer's coefficient for the population "all high school students residing in Japan" is 0. If that's true, then the probability that χ_0^2 obtained from the 300 people who have been chosen randomly is 5.9915 or more is 0.05.

FIGURE 7-2: PROBABILITY THAT χ_0^2 IS 5.9915 OR MORE



This is clear from the table of chi-square distribution on page 103.

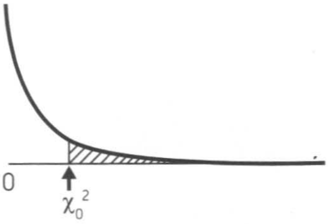
χ_0^2 for this exercise has already been calculated on page 132. It is 8.0091. True, this figure has been calculated based on data from 300 randomly chosen people, but doesn't this seem too large? Taking into consideration the comment on page 132, isn't it natural to assume that the Cramer's coefficient for the population "all high school students residing in Japan" is greater than 0?

Remember that the process for a chi-square test of independence (not limited to this exercise) goes like this:

1. Assume a null hypothesis that "the Cramer's coefficient for the population is 0" for the time being.
2. Calculate χ_0^2 from the sample data.
3. If χ_0^2 is too large, reject the null hypothesis and conclude that "the Cramer's coefficient for the population is greater than 0."

As χ_0^2 becomes larger, the probability shown as the shaded area in Figure 7-3 naturally becomes smaller.

FIGURE 7-3: PROBABILITY IN CORRESPONDENCE TO χ_0^2

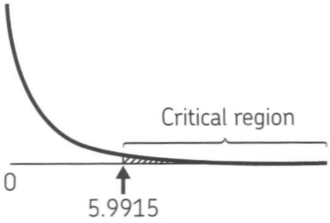


In chi-square tests of independence, if the probability shown as the shaded area in Figure 7-3 is less than or equal to the value called the *significance level*, reject the null hypothesis and conclude that "the Cramer's coefficient for the population is greater than 0." In general, the significance level (also called the *alpha value* and expressed by the symbol α) is considered to be 0.05 or 0.01.

It is up to the analyst which significance level to use. Suppose we decide to use 0.05 as the significance level in this case. The significance level is in fact the probability expressed as the shaded area in Figure 7-3.

The shaded area in Figure 7-4 is called the *critical region*.

FIGURE 7-4: CRITICAL REGION
(WHEN SIGNIFICANCE LEVEL IS 0.05)



ANSWER

Step 1

Define the population.

The population is:

ALL HIGH SCHOOL STUDENTS RESIDING IN JAPAN

In this exercise, the population was defined as "all high school students residing in Japan." Thus, in this particular exercise, step 1 is unnecessary.

However, for "Tests of difference between population ratios" in the table on page 149, the populations in question are "voters residing in urban areas" and "voters residing in rural areas." Where are the urban areas exactly? Are they Tokyo and Osaka? Or are they the capitals of the prefectures? This must be specified by the analyst.

I repeat: When you are actually doing a hypothesis test, you must determine the population. No matter which hypothesis test you are trying to carry out, you must not fail to properly define the population.

Otherwise, you might fall into a situation in which you are lost, wondering, "What was I trying to estimate?" Lots of statisticians fall into traps like this. Take great care about this point.

Step 2

Set up a null hypothesis and an alternative hypothesis.

The null hypothesis is: "The Cramer's coefficient for the population is 0. In other words, sex and desired way of being asked out are not correlated."

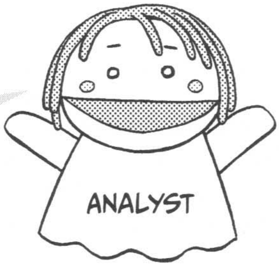
The alternative hypothesis is: "The Cramer's coefficient for the population is greater than 0. In other words, sex and desired way of being asked out are correlated."

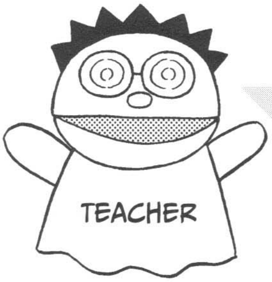
An explanation of null hypotheses and alternative hypotheses is given on page 170.

Step 3

Choose which hypothesis test to do.

I am going to do a chi-square test of independence.





This exercise asks you to do a chi-square test of independence. So in this particular exercise, step 3 is unnecessary.

(When you are actually doing a hypothesis test and not an exercise, you must select the hypothesis test suitable for the objective of analysis on your own.)

Step 4

Determine the significance level.

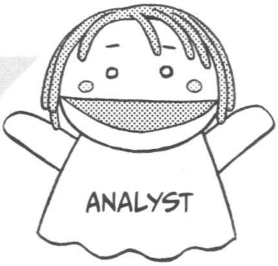
I will use 0.05 as the significance level.

The exercise assigns 0.05 as the significance level, so in this particular exercise, step 4 is unnecessary. When you are actually doing a hypothesis test and not an exercise, you must determine the significance level. As mentioned earlier, normally either 0.05 or 0.01 is used. The smaller the P-value computed from the sample data, the stronger the evidence is against the null hypothesis. In general, the symbol α is used to express the significance level (alpha value).

Step 5

Calculate the test statistic from the sample data.

I am trying to do a chi-square test of independence. Thus the test statistic is Pearson's chi-square test statistic (χ_0^2). The value of χ_0^2 for this exercise has already been calculated on page 132: $\chi_0^2 = 8.0091$.



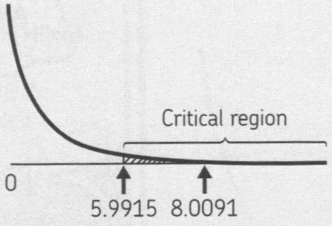
The test statistic is obtained from a function that calculates a single value from the sample data. Different kinds of hypothesis tests have different test statistics. As mentioned above, the value for a test of independence is χ_0^2, and in the case of tests of correlation (see page 149), the test statistic is as below.

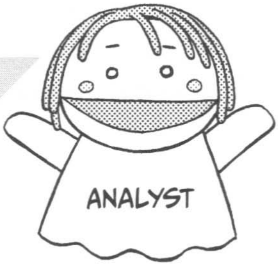
$$\frac{\text{correlation coefficient}^2 \times \sqrt{\text{number of values} - 2}}{1 - \sqrt{\text{correlation coefficient}^2}}$$

Step 6

Determine whether or not the test statistic from step 5 is in the critical region.

Pearson's chi-square test statistic (χ_0^2) is 8.0091. As the significance level (α) is 0.05, the critical region is 5.9915 or above, according to the table of chi-square distribution on page 103. As shown in the chart below, the test statistic is within the critical region.



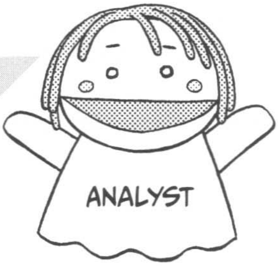


The critical region changes depending on the significance level (α). If α in this exercise was 0.01 instead of 0.05, the critical region would be 9.2104 or above, according to the table of chi-square distribution on page 103.

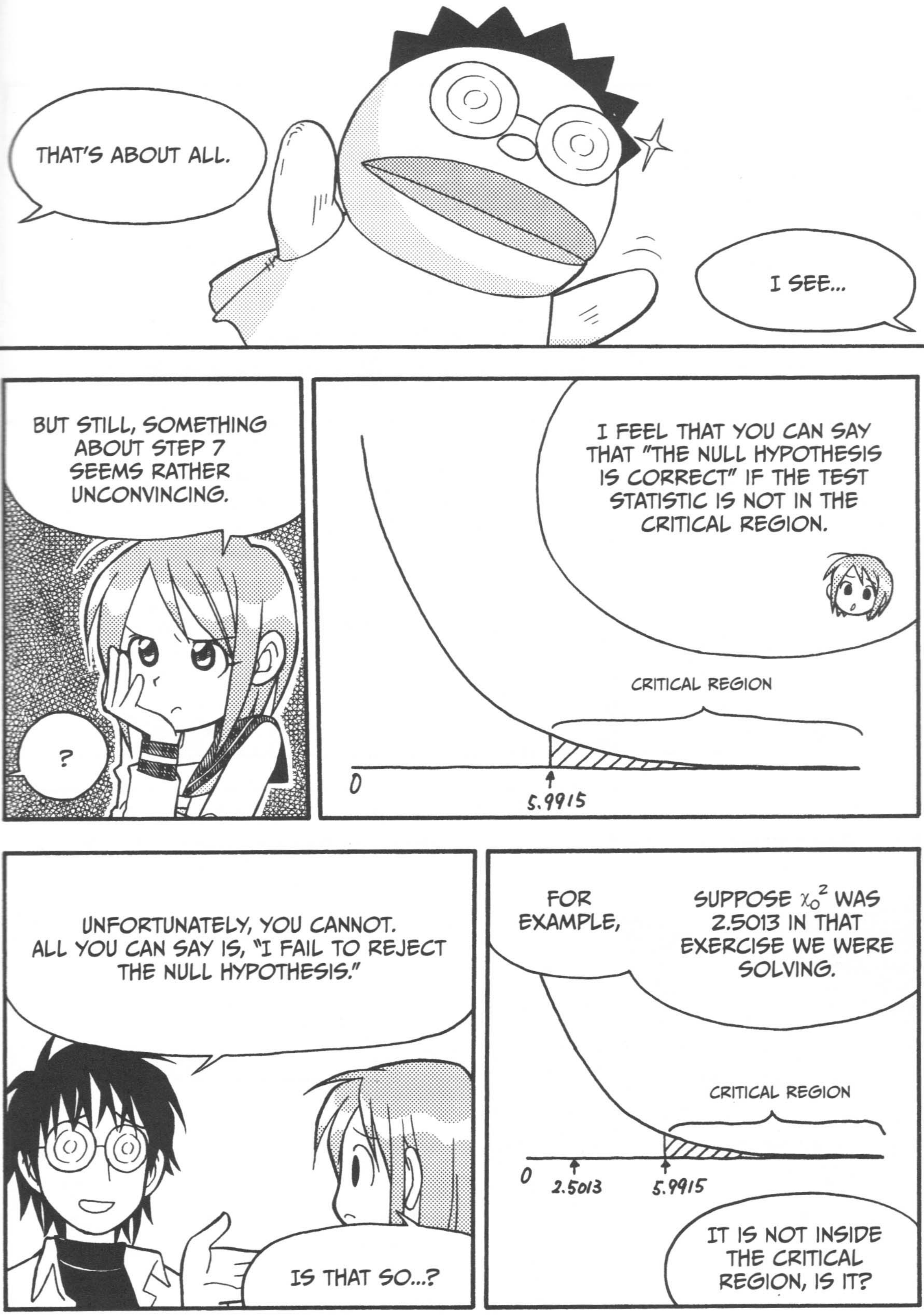
Step 7

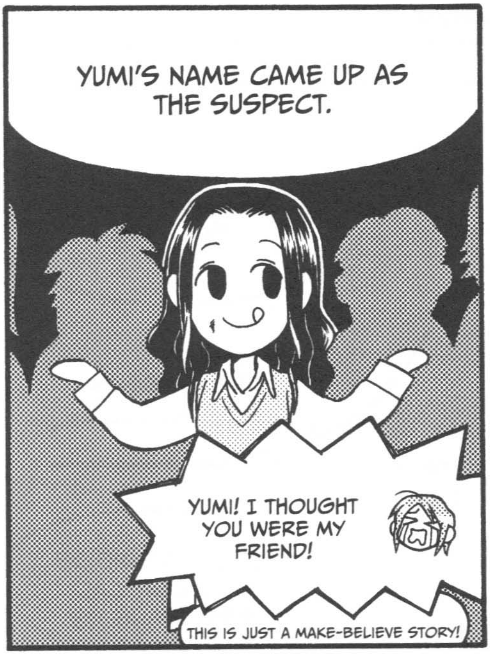
If the test statistic is in the critical region in step 6, you reject the null hypothesis. If not, you fail to reject the null hypothesis. In this case, the test statistic was in the critical region.

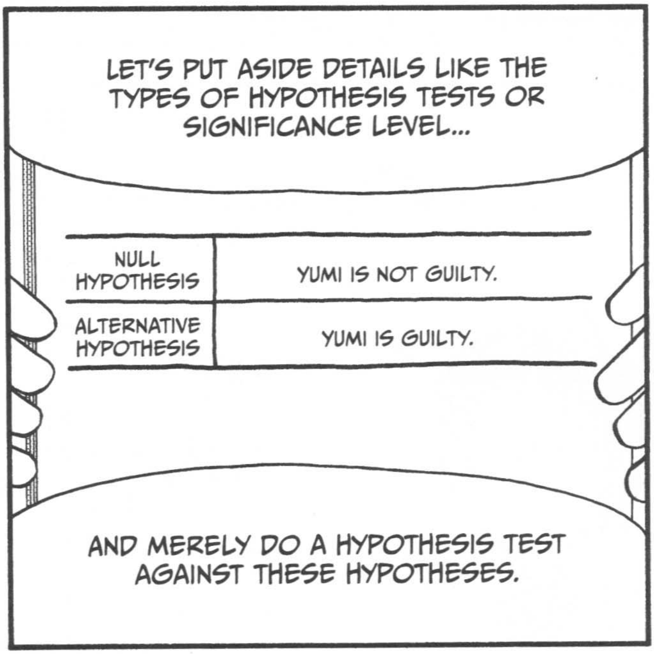
Thus the alternative hypothesis, "the Cramer's coefficient for the population is greater than 0," is correct!



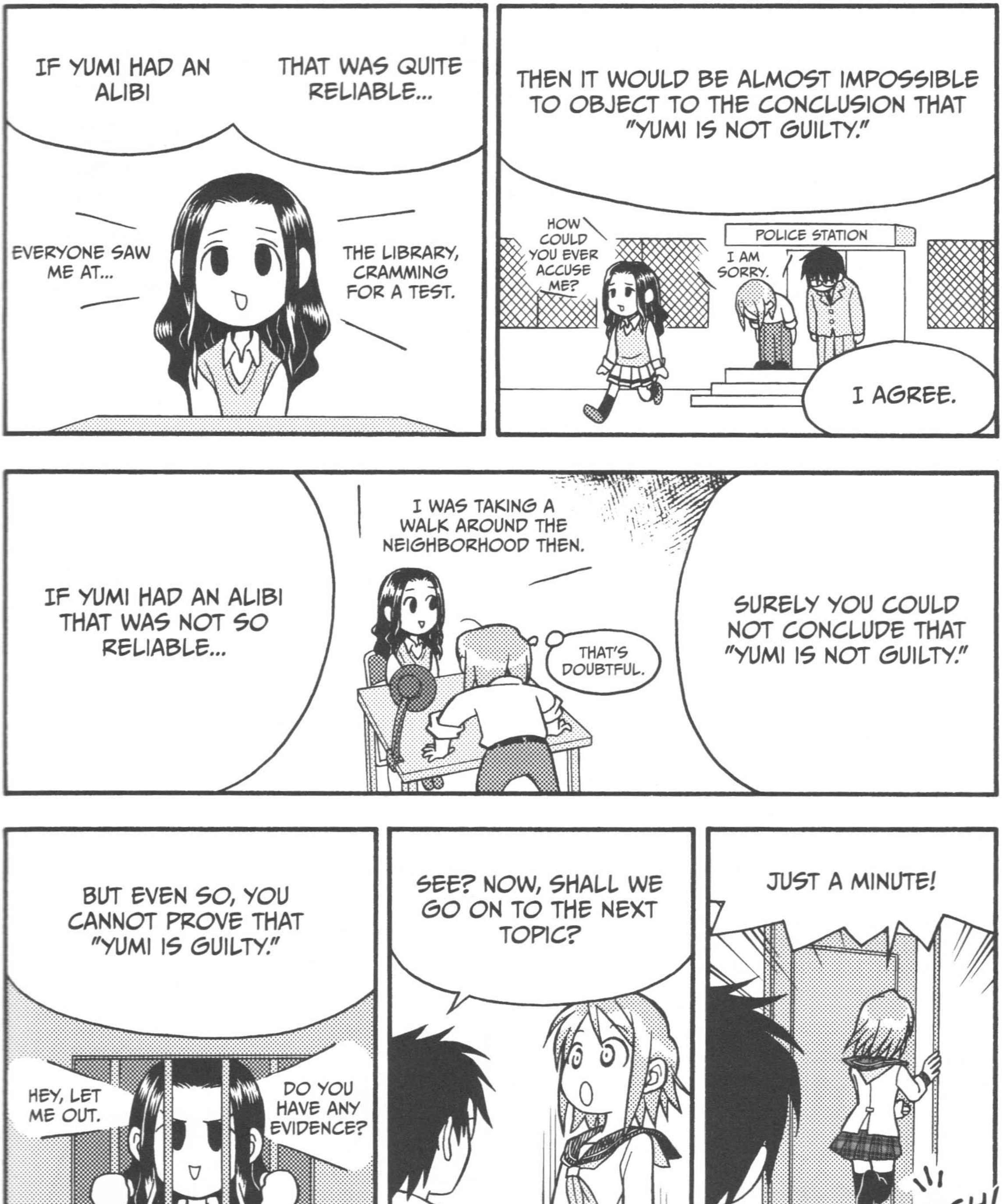
You cannot conclude that the alternative hypothesis is absolutely correct in a hypothesis test, even if the test statistic is within the critical region. The only conclusion you can make is, "I would like to say that the alternative hypothesis is 'absolutely' correct . . . but there is, at most, a ($\alpha \times 100$)% possibility that the null hypothesis is correct."


THAT'S ABOUT ALL.
I SEE...
BUT STILL, SOMETHING ABOUT STEP 7 SEEMS RATHER UNCONVINCING.
?
I FEEL THAT YOU CAN SAY THAT "THE NULL HYPOTHESIS IS CORRECT" IF THE TEST STATISTIC IS NOT IN THE CRITICAL REGION.
CRITICAL REGION
0
5.9915
UNFORTUNATELY, YOU CANNOT. ALL YOU CAN SAY IS, "I FAIL TO REJECT THE NULL HYPOTHESIS."
IS THAT SO...?
FOR EXAMPLE,
SUPPOSE χ_0^2 WAS 2.5013 IN THAT EXERCISE WE WERE SOLVING.
CRITICAL REGION
0
2.5013
5.9915
IT IS NOT INSIDE THE CRITICAL REGION, IS IT?

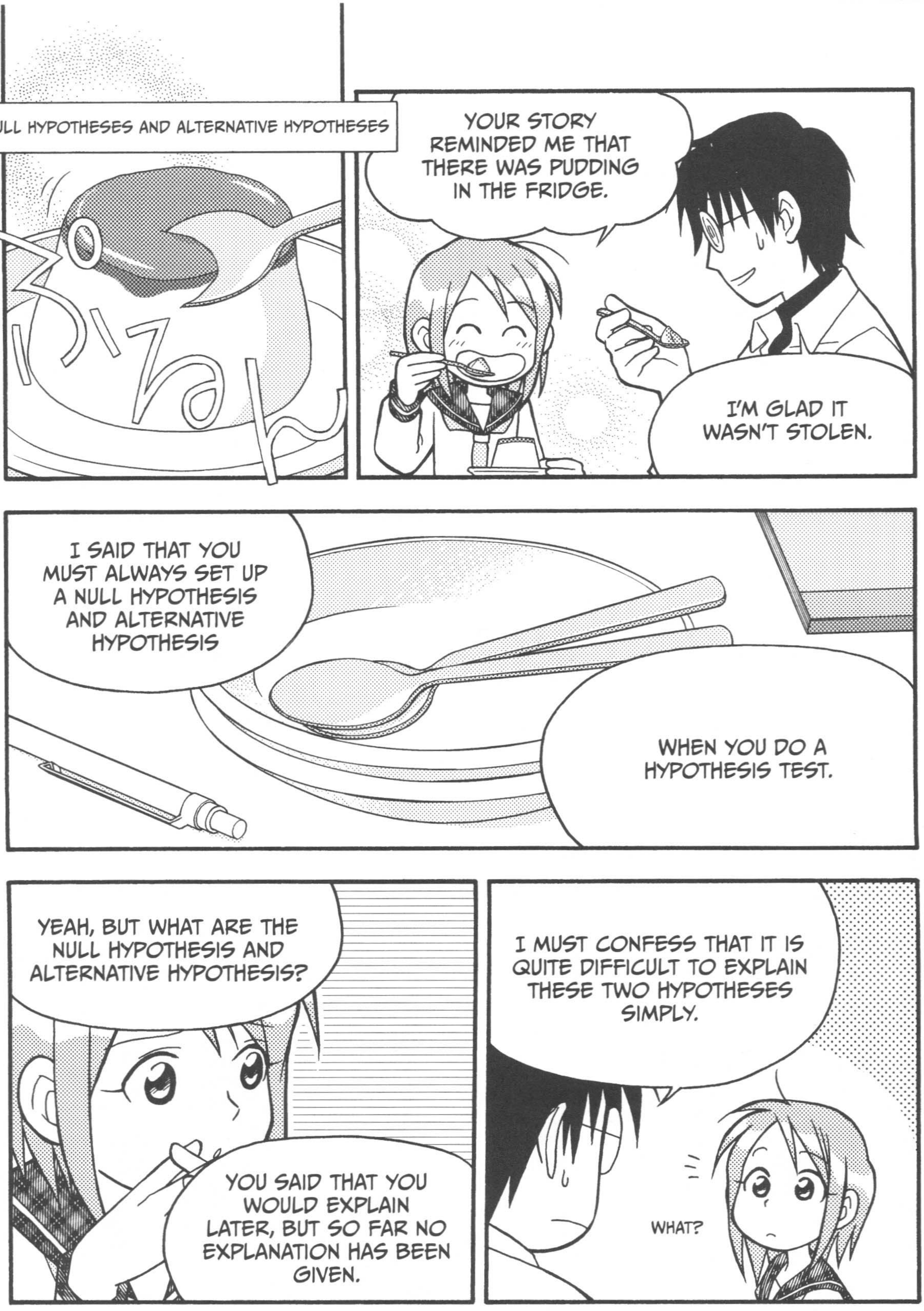


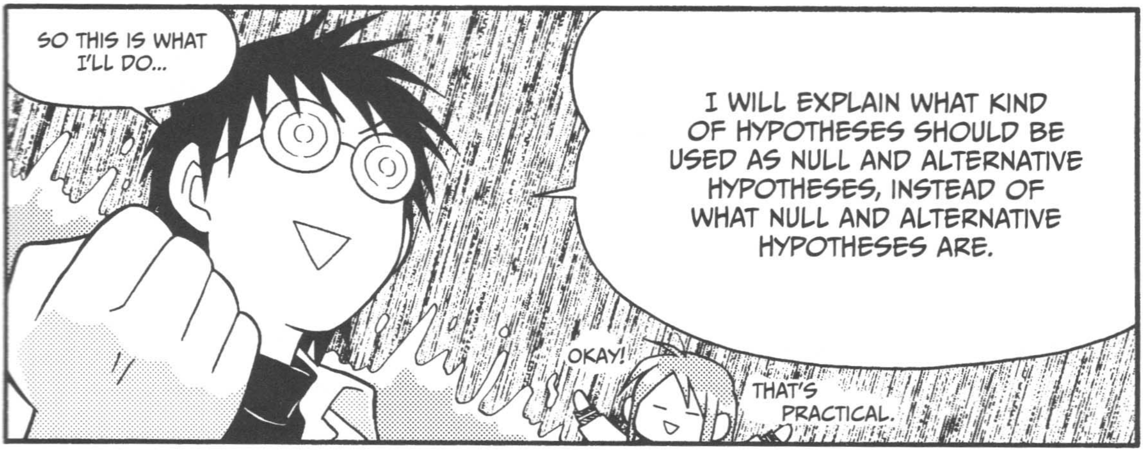


NULL HYPOTHESIS	YUMI IS NOT GUILTY.
ALTERNATIVE HYPOTHESIS	YUMI IS GUILTY.


IF YUMI HAD AN ALIBI
THAT WAS QUITE RELIABLE...
EVERYONE SAW ME AT...
THE LIBRARY, CRAMMING FOR A TEST.
THEN IT WOULD BE ALMOST IMPOSSIBLE TO OBJECT TO THE CONCLUSION THAT "YUMI IS NOT GUILTY."
POLICE STATION
HOW COULD YOU EVER ACCUSE ME?
I AM SORRY.
I AGREE.
IF YUMI HAD AN ALIBI THAT WAS NOT SO RELIABLE...
I WAS TAKING A WALK AROUND THE NEIGHBORHOOD THEN.
THAT'S DOUBTFUL.
SURELY YOU COULD NOT CONCLUDE THAT "YUMI IS NOT GUILTY."
BUT EVEN SO, YOU CANNOT PROVE THAT "YUMI IS GUILTY."
HEY, LET ME OUT.
DO YOU HAVE ANY EVIDENCE?
THAT'S TRUE.
SEE? NOW, SHALL WE GO ON TO THE NEXT TOPIC?
OH!
JUST A MINUTE!
DASH!
?

3. NULL HYPOTHESES AND ALTERNATIVE HYPOTHESES

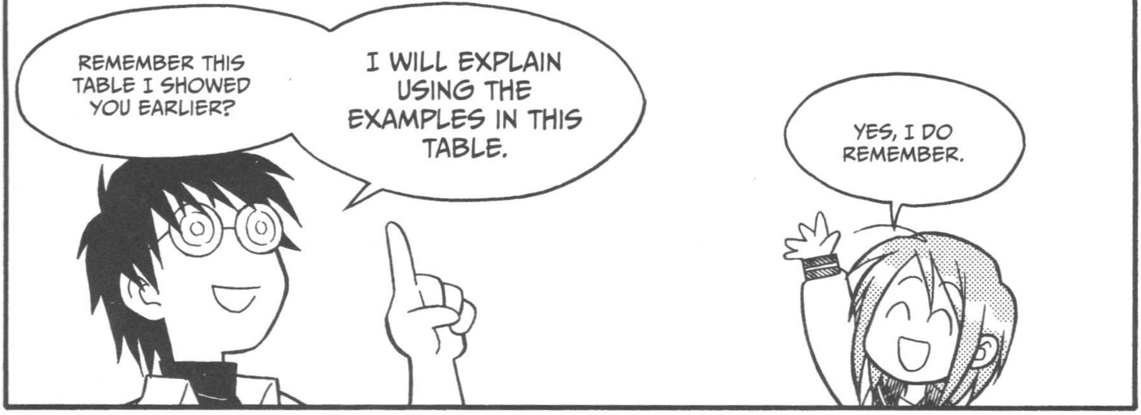




EXAMPLES OF HYPOTHESIS TESTS

Name	**Example of use**
Tests of independence	Estimates whether the value of the Cramer's coefficient for sex and desired way of being asked out is zero for a population
Tests of correlation ratio	Estimates whether the value of the correlation ratio for favorite fashion brand and age is zero for a population
Tests of correlation	Estimates whether the correlation coefficient for amount spent on makeup and amount spent on clothes is zero for a population
Tests of difference between population means	Estimates whether allowances are different between high school girls in Tokyo and Osaka*
Tests of difference between population ratios	Estimates whether the approval rating of cabinet *X* is different between voters residing in urban areas and rural areas*

* Note that two populations are being considered.



TESTS OF INDEPENDENCE

Null hypothesis	The Cramer's coefficient for sex and desired way of being asked out is 0 for a population.
Alternative hypothesis	The Cramer's coefficient for sex and desired way of being asked out is greater than 0 for a population.

TESTS OF CORRELATION RATIO

Null hypothesis	The correlation ratio for favorite fashion brand and age is 0 for a population.
Alternative hypothesis	The correlation ratio for favorite fashion brand and age is greater than 0 for a population.

TESTS OF CORRELATION

Null hypothesis	The correlation coefficient for amount spent on makeup and amount spent on clothes is 0 for a population.
Alternative hypothesis	The correlation coefficient for amount spent on makeup and amount spent on clothes is not 0 for a population. or The correlation coefficient for amount spent on makeup and amount spent on clothes is greater than 0 for a population. or The correlation coefficient for amount spent on makeup and amount spent on clothes is less than 0 for a population.

TESTS OF DIFFERENCE BETWEEN POPULATION MEANS

Null hypothesis	The allowances of high school girls in Tokyo and Osaka are the same.
Alternative hypothesis	The allowances of high school girls in Tokyo and Osaka are not the same. or The allowances of high school girls in Tokyo are larger than those of high school girls in Osaka. or The allowances of high school girls in Tokyo are smaller than those of high school girls in Osaka.

TESTS OF DIFFERENCE BETWEEN POPULATION RATIOS

Null hypothesis	The approval ratings of cabinet *X* for voters residing in urban areas and rural areas are the same.
Alternative hypothesis	The approval ratings of cabinet *X* for voters residing in urban areas and rural areas are not the same. or The approval rating of cabinet *X* for voters residing in urban areas is higher than that of voters residing in rural areas. or The approval rating of cabinet *X* for voters residing in urban areas is lower than that of voters residing in rural areas.


FOLLOWING ALONG SO FAR?
I HOPE YOU NOW UNDERSTAND WHY "THE CRAMER'S COEFFICIENT FOR THE POPULATION IS 'ALMOST' ZERO" IS NOT THE ONLY NULL HYPOTHESIS, AND THERE EXIST SOME NULL HYPOTHESES THAT SEEM DIFFICULT TO PROVE, SUCH AS "THE CRAMER'S COEFFICIENT FOR THE POPULATION IS ZERO."
YES I DO. SO MANY OF THEM ARE VERY EXTREME.
ALSO, DID YOU NOTICE THAT NULL HYPOTHESES ARE POSITIVE AND USE PHRASES LIKE "...IS..." OR "...ARE THE SAME"?
ALTERNATIVE HYPOTHESES ARE NEGATIVE, WITH PHRASES LIKE "...IS NOT..." OR "...ARE NOT THE SAME."
THAT'S TRUE.
THIS IS HOW YOU SHOULD THINK ABOUT THIS MATTER.
USE A HYPOTHESIS THAT SEEMS DIFFICULT TO PROVE AND IS POSITIVE FOR THE NULL HYPOTHESIS.
THAT'S CONVINCING.
THEN SET A HYPOTHESIS THAT IS OPPOSITE TO THE NULL HYPOTHESIS AS THE ALTERNATIVE HYPOTHESIS.

4. P-VALUE AND PROCEDURE FOR HYPOTHESIS TESTS


YOU MEAN THE PART THAT'S SHADED?
UNTIL PERSONAL COMPUTERS BECAME POPULAR, IT WAS AWFULLY DIFFICULT TO CALCULATE THE P-VALUE.
THAT WAS AROUND THE BEGINNING OF THE 1990S.
OH, REALLY...
FOR THAT REASON, THEY USED TO USE (1)...
...FOR MOST OF THE CONCLUSIONS IN HYPOTHESIS TESTS.
WHAT ABOUT NOW?
THE P-VALUE CAN BE CALCULATED EASILY BY USING MICROSOFT EXCEL AND OTHER SOFTWARE. SO (2) IS STARTING TO BE USED MORE OFTEN.
THAT'S SATISFYING.
THE PROCESS FOR (2) IS DIFFERENT FROM THE PROCESS I JUST EXPLAINED FOR (1), SO...
"THE PUPPET STATISTICIAN IS GOING TO DO THE ANALYSIS ONCE AGAIN!"
TEE HEE HEE!
STOP! YOUR FAKE GIRL VOICE IS DISTURBING!

Step 6p

Determine whether or not the P-value corresponding to the test statistic obtained in step 5 is smaller than the significance level.

The significance level is 0.05. Since Pearson's chi-square test statistic (χ_0^2, which is the test statistic in this case) is 8.0091, the P-value is 0.0182.

$0.0182 < 0.05$

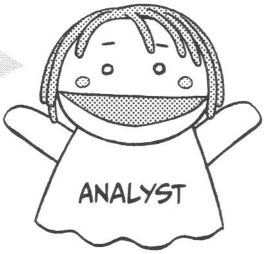
Thus the P-value is smaller.

As mentioned before, you can calculate the P-value using Excel (though this depends on what type of hypothesis test you are doing). See page 208 for details.

Step 7p

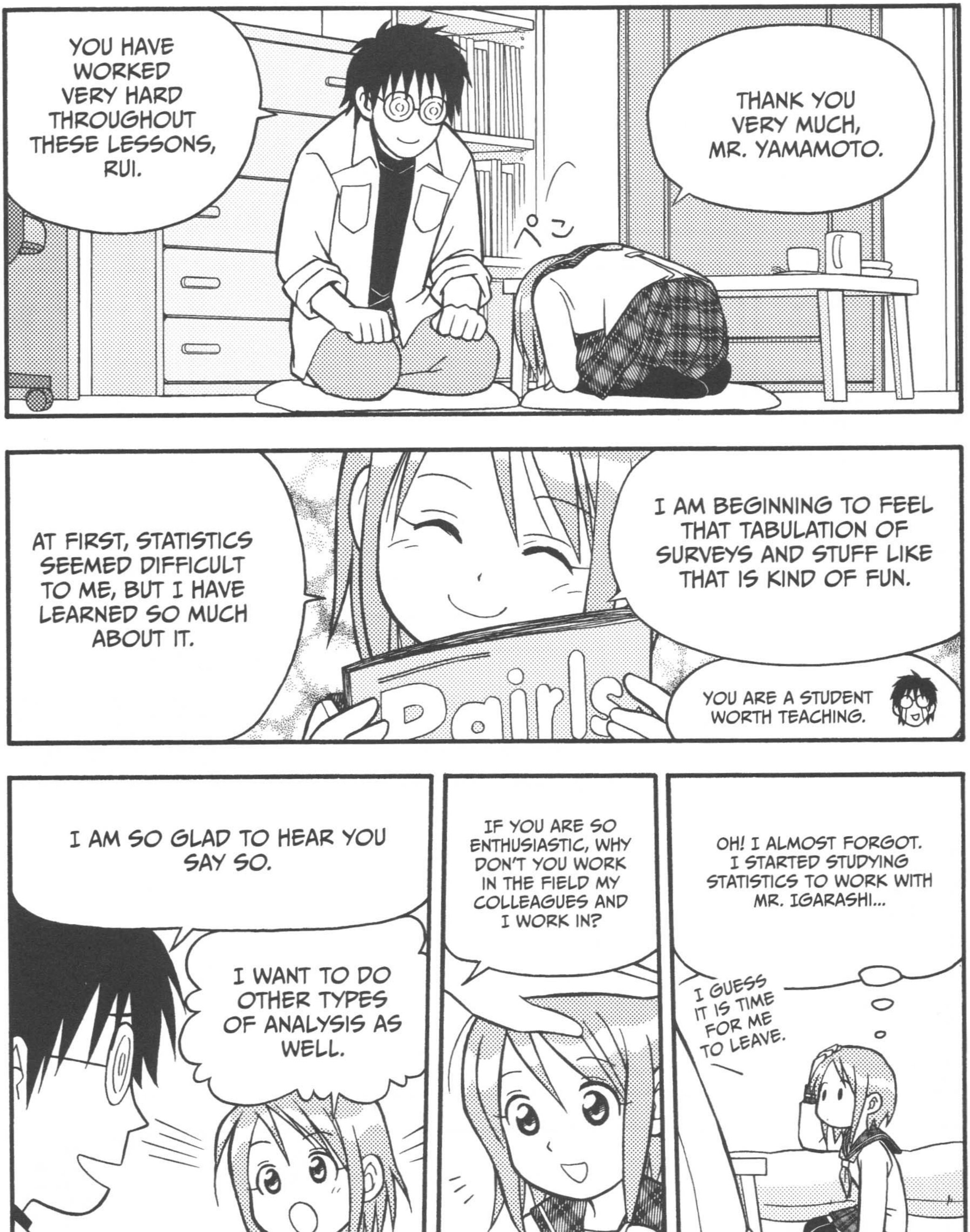
If the P-value is smaller than the significance level in step 6p, you reject the null hypothesis. If not, you fail to reject the null hypothesis.

The P-value was smaller than the significance level. Therefore, you conclude in favor of the alternative hypothesis, "the Cramer's coefficient for the population is greater than 0."



Even if the P-value was smaller than the significance level, you cannot really conclude that the alternative hypothesis is "absolutely" correct in a hypothesis test. The only conclusion you can make is: "I would like to say that the alternative hypothesis is 'absolutely' correct . . . but there is a ($\alpha \times 100$)% possibility that the null hypothesis is correct."


WHY, THAT'S VERY SIMILAR TO (1).
I THINK I UNDERSTAND, FOR THE MOST PART.
YOU ARE SO SMART! I AM PROUD OF YOU.
AM I?
NOW, RUI, RECALL THE STORY ABOUT THE PUDDING IN (1).
IT SEEMS THAT YOU CAN CONCLUDE THAT THE NULL HYPOTHESIS IS CORRECT IF THE P-VALUE IS LARGER THAN THE SIGNIFICANCE LEVEL, BUT THAT'S IMPOSSIBLE.
ALL YOU CAN DO IS...
FAIL TO REJECT THE NULL HYPOTHESIS!
SMILE
TEARS
(YOU'VE UNDERSTOOD AT LAST!)


YOU HAVE WORKED VERY HARD THROUGHOUT THESE LESSONS, RUI.
ペコ
THANK YOU VERY MUCH, MR. YAMAMOTO.
AT FIRST, STATISTICS SEEMED DIFFICULT TO ME, BUT I HAVE LEARNED SO MUCH ABOUT IT.
Pairls
I AM BEGINNING TO FEEL THAT TABULATION OF SURVEYS AND STUFF LIKE THAT IS KIND OF FUN.
YOU ARE A STUDENT WORTH TEACHING.
I AM SO GLAD TO HEAR YOU SAY SO.
I WANT TO DO OTHER TYPES OF ANALYSIS AS WELL.
IF YOU ARE SO ENTHUSIASTIC, WHY DON'T YOU WORK IN THE FIELD MY COLLEAGUES AND I WORK IN?
OH! I ALMOST FORGOT. I STARTED STUDYING STATISTICS TO WORK WITH MR. IGARASHI...
I GUESS IT IS TIME FOR ME TO LEAVE.


I MUST GO SEE MR. IGARASHI.
WHAT? IGARASHI? HE'S GONE...
...ON VACATION. HE IS ON HIS HONEYMOON.
WHAT? HE'S MARRIED?!
THAT CAN'T BE TRUE... I HAVE BEEN STUDYING STATISTICS ALL THIS TIME IN VAIN...!?
ガーン
ガーン
RUI? I THOUGHT YOU WERE STUDYING FOR YOUR OWN SAKE..!
SHUT UP!
HEY, BE CAREFUL!
CRASH
SORRY...


ARE YOU ALL RIGHT?
I DIDN'T KNOW YOU WERE SO HANDSOME!


MR. YAMAMOTO, PLEASE DO KEEP TEACHING ME THIS AND THAT!
AND SO THEIR LESSONS WILL CONTINUE...
...MAYBE...MAYBE NOT.

5. TESTS OF INDEPENDENCE AND TESTS OF HOMOGENEITY

There is a hypothesis test very similar to a test of independence called a *test of homogeneity*. Below is an example of a test of homogeneity. As you read it, think about how it is different from a test of independence.

EXAMPLE

P-Girls Magazine published an article titled, "We Asked 300 High School Students, 'How Would You Like to Be Asked Out?'" The choices were phone, e-mail, or face to face.

> **HYPOTHESIS: THE RATIO OF PHONE TO E-MAIL TO FACE-TO-FACE IS DIFFERENT BETWEEN HIGH SCHOOL GIRLS AND BOYS.**

To find out if this hypothesis is true or not, a journalist actually conducted a survey by randomly choosing respondents from each of the two groups, "all high school girls residing in Japan" and "all high school boys residing in Japan." The table below is the result.

Respondent	Desired way of being asked out	Age	Sex
1	Face to face	17	Female
...	...	...	...
148	E-mail	16	Female
149	Phone	15	Male
...	...	...	...
300	E-mail	18	Male

The cross tabulation of sex and desired way of being asked out is the table below.

		Desired way of being asked out			Sum
		Phone	E-mail	Face to face	
Sex	Female	34	61	53	148
	Male	38	40	74	152
Sum		72	101	127	300

Estimate whether or not the hypothesis stated above is correct using a test of homogeneity. Use 0.05 as the significance level.

PROCEDURE

Step 1	Define the population.	The population in this case is "all high school girls residing in Japan" and "all high school boys residing in Japan."
Step 2	Set up a null hypothesis and an alternative hypothesis.	The null hypothesis is "the ratio of phone to e-mail to face to face is the same for high school girls and boys." The alternative hypothesis is "the ratio of phone to e-mail to face to face is different between high school girls and boys."
Step 3	Choose which hypothesis test to do.	A test of homogeneity will be applied.
Step 4	Determine the significance level.	The significance level is 0.05.
Step 5	Calculate the test statistic from the sample data.	A test of homogeneity is being used in this exercise. Therefore, the test statistic is Pearson's chi-square test statistic. The value of χ_0^2 in this exercise has already been calculated on page 132. $\chi_0^2 = 8.0091$ Pearson's chi-square test statistic (χ_0^2) in this exercise follows a chi-square distribution of degrees of freedom $(2 - 1) \times (3 - 1) = 1 \times 2 = 2$, if the null hypothesis is true.
Step 6	Determine whether the test statistic in step 5 is in the critical region.	The test statistic χ_0^2 is 8.0091. Since the significance level is 0.05, the critical region is 5.9915 or more, according to the table of chi-square distribution on page 103. The test statistic is within the critical region.
Step 7	If the test statistic is in the critical region in step 6, reject the null hypothesis and conclude in favor of the alternative. If not, fail to reject the null hypothesis.	The test statistic was within the critical region. Thus, you conclude in favor of the alternative hypothesis, "the ratio of phone to e-mail to face to face is different between high school girls and boys."

Don't you think that both the exercise and procedure are quite similar to those for a test of independence? Let's now look at the differences between tests of independence and tests of homogeneity. There are three things to note.

First, the population defined is different. There is only one population ("all high school students residing in Japan") in the former. In the latter, there are two populations: "all high school girls residing in Japan" and "all high school boys residing in Japan."

Also, the hypotheses are different. In the former,

Null hypothesis	The Cramer's coefficient for the population is 0. In other words, sex and desired way of being asked out are not correlated.
Alternative hypothesis	The Cramer's coefficient for the population is greater than 0. In other words, sex and desired way of being asked out are correlated.

In the latter,

Null hypothesis	The ratio of phone to e-mail to face to face is the same for high school girls and boys.
Alternative hypothesis	The ratio of phone to e-mail to face to face is different between high school girls and boys.

Finally, the order of procedure is different. In the former, the hypothesis is set after the data is collected, whereas the hypothesis is set before collecting the data in the latter.

As confirmed in the previous paragraph, tests of independence and tests of homogeneity have obvious differences. However, in practice, people tend to do tests of homogeneity when they are actually intending to do tests of independence, or vice versa. Be careful.

6. HYPOTHESIS TEST CONCLUSIONS

Up to this point, we have expressed the conclusion of a hypothesis test as follows:

> **IF THE TEST STATISTIC IS IN THE CRITICAL REGION, YOU CAN CONCLUDE, "I REJECT THE NULL HYPOTHESIS." IF NOT, YOU CONCLUDE, "I FAIL TO REJECT THE NULL HYPOTHESIS."**

But there are other ways to express the conclusions of hypothesis tests. They are summarized below.

TABLE 7-4: EXPRESSIONS OF HYPOTHESIS TEST CONCLUSIONS

When the test statistic is in the critical region	**When the test statistic is not in the critical region**
• Conclude in favor of the alternative hypothesis • Conclude that the result is statistically significant • Reject the null hypothesis	• Fail to reject the null hypothesis • Conclude that the result is not statistically significant • Accept the null hypothesis

The expressions "it is statistically significant" and "it is not statistically significant" seem to be popular in introductions to statistics. So why did we use an unpopular expression on purpose? I recognize that many beginners to hypothesis tests use the expression "it is significant" without actually understanding the meaning of the phrase. They seem to be merely confirming the test statistic or P-value. If you do not set a proper null and alternative hypothesis, the meaning of *significant* will be ambiguous. Beginners' definitions of their populations are frequently unclear as well.

I used to think I shouldn't be so strict with beginners. But it's impossible to make an accurate conclusion with uncertain null and alternative hypotheses. So in this book, I use the expressions "reject the null hypothesis" and "fail to reject the null hypothesis" so that you will get into the habit of thinking hard about your hypotheses.

EXERCISE AND ANSWER

EXERCISE

The table below is the same as the cross tabulation found on page 138.

		Preference for coffee or tea		Sum
		Coffee	Tea	
Type of food often ordered	Japanese	43	33	76
	European	51	53	104
	Chinese	29	41	70
Sum		123	127	250

Using a chi-square test of independence, estimate if the Cramer's coefficient for type of food often ordered and preference for coffee or tea in the population "people of age 20 or older residing in Japan" is greater than 0. This is the same as estimating whether there is a correlation between type of food often ordered and preference for coffee or tea. Use 0.01 as the significance level.

ANSWER

Step 1	Define the population.	The population in this case is "people of age 20 or older residing in Japan."
Step 2	Set up a null hypothesis and an alternative hypothesis.	The null hypothesis is "type of food often ordered and preference for coffee or tea are not correlated." The alternative hypothesis is "type of food often ordered and preference for coffee or tea are correlated."
Step 3	Choose which hypothesis test to do.	A chi-square test of independence will be applied.
Step 4	Determine the significance level.	The significance level is 0.01.
Step 5	Calculate the test statistic from the sample data.	A chi-square test of independence is being used in this exercise. Therefore, the test statistic is Pearson's chi-square test statistic (χ_0^2). The value of χ_0^2 in this exercise has already been calculated on page 141. $\chi_0^2 = 3.3483$
Step 6	Determine whether the test statistic obtained in step 5 is in the critical region.	The test statistic χ_0^2 is 3.3483. Because the significance level (α) is 0.01, the critical region is 9.2104 or above, according to the table of chi-square distribution on page 103. The test statistic is not within the critical region.
Step 7	If the test statistic is in the critical region in step 6, reject the null hypothesis. If not, fail to reject the null hypothesis.	The test statistic was not within the critical region. Thus, the null hypothesis "type of food often ordered and preference for coffee or tea are not correlated" cannot be rejected.

SUMMARY

- A *hypothesis test* is an analysis technique used to estimate whether the analyst's hypothesis about the population is correct using the sample data.
- The formal name for a hypothesis test is *statistical hypothesis testing.*
- Test statistics are obtained from a function that calculates a single value from the sample data.
- In general, 0.05 or 0.01 is used as the significance level.
- The *critical region* is an area that corresponds to the significance level (also called the *alpha value* and expressed by the symbol α).
- A *chi-square test of independence* is an analysis technique used to estimate whether the Cramer's coefficient for a population is 0. It can also be said that it is an analysis technique used to estimate whether the two variables in a cross tabulation are correlated.
- If the Cramer's coefficient for a population is 0, Pearson's chi-square test statistic follows a chi-square distribution.
- The *P-value* in a test of independence is a probability that gives a Pearson's chi-square test statistic equal to or greater than the value earned in the case when the null hypothesis is true.
- When making a conclusion in a hypothesis test, there are two bases of judgment:

  1. Whether the test statistic is in the critical region
  2. Whether the P-value is smaller than the significance level

- The process of analysis in any hypothesis test is the same as the process for the test of independence or any other kind of test. The actual procedure is:

Step 1	Define the population.
Step 2	Set up a null hypothesis and an alternative hypothesis.
Step 3	Choose which hypothesis test to do.
Step 4	Determine the significance level.
Step 5	Calculate the value of the test statistic from the sample data.
Step 6	Determine whether the test statistic obtained in step 5 is in the critical region.
Step 7	If the test statistic is in the critical region in step 6, reject the null hypothesis. If not, fail to reject the null hypothesis.
Step 6p	Determine whether the P-value corresponding to the test statistic obtained in step 5 is smaller than the significance level.
Step 7p	If the P-value is smaller than the significance level in step 6p, reject the null hypothesis. If not, fail to reject the null hypothesis.

LET'S CALCULATE USING EXCEL

This appendix contains instructions for calculating various statistics using Microsoft Excel. You'll learn how to do the following things:

1. Make a frequency table
2. Calculate arithmetic mean, median, and standard deviation
3. Make a cross tabulation
4. Calculate the standard score and the deviation score
5. Calculate the probability of the standard normal distribution
6. Calculate the point on the horizontal axis of the chi-square distribution
7. Calculate the correlation coefficient
8. Perform tests of independence

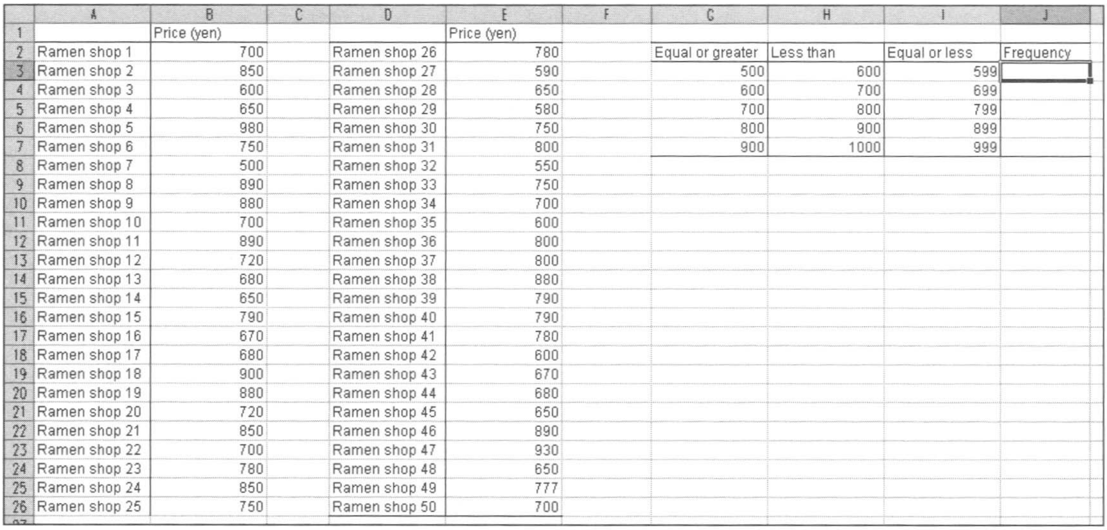
You can download these Excel files and follow along (get them at *http://www.nostarch.com/mg_statistics.htm*). Readers who are not familiar with Excel should try "Calculating Arithmetic Mean, Median, and Standard Deviation" on page 195 first.

1. MAKING A FREQUENCY TABLE

This exercise uses the ramen restaurant prices on page 33.

Step 1

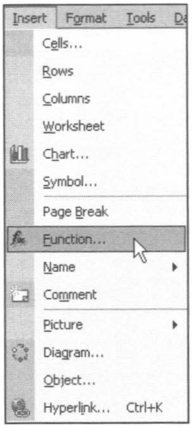
Select cell J3.



	A	B	C	D	E	F	G	H	I	J
1		Price (yen)			Price (yen)					
2	Ramen shop 1	700		Ramen shop 26	780		Equal or greater	Less than	Equal or less	Frequency
3	Ramen shop 2	850		Ramen shop 27	590		500	600	599	
4	Ramen shop 3	600		Ramen shop 28	650		600	700	699	
5	Ramen shop 4	650		Ramen shop 29	580		700	800	799	
6	Ramen shop 5	980		Ramen shop 30	750		800	900	899	
7	Ramen shop 6	750		Ramen shop 31	800		900	1000	999	
8	Ramen shop 7	500		Ramen shop 32	550					
9	Ramen shop 8	890		Ramen shop 33	750					
10	Ramen shop 9	880		Ramen shop 34	700					
11	Ramen shop 10	700		Ramen shop 35	600					
12	Ramen shop 11	890		Ramen shop 36	800					
13	Ramen shop 12	720		Ramen shop 37	800					
14	Ramen shop 13	680		Ramen shop 38	880					
15	Ramen shop 14	650		Ramen shop 39	790					
16	Ramen shop 15	790		Ramen shop 40	790					
17	Ramen shop 16	670		Ramen shop 41	780					
18	Ramen shop 17	680		Ramen shop 42	600					
19	Ramen shop 18	900		Ramen shop 43	670					
20	Ramen shop 19	880		Ramen shop 44	680					
21	Ramen shop 20	720		Ramen shop 45	650					
22	Ramen shop 21	850		Ramen shop 46	890					
23	Ramen shop 22	700		Ramen shop 47	930					
24	Ramen shop 23	780		Ramen shop 48	650					
25	Ramen shop 24	850		Ramen shop 49	777					
26	Ramen shop 25	750		Ramen shop 50	700					

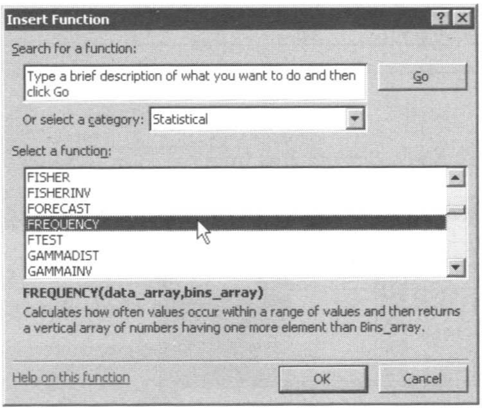
Step 2

Select **Insert ▸ Function**.



Step 3

Select **Statistical** from the category dropdown menu, and then select **FREQUENCY** as the name of the function.



Insert Function

Search for a function:

Type a brief description of what you want to do and then click Go

Or select a category: Statistical

Select a function:

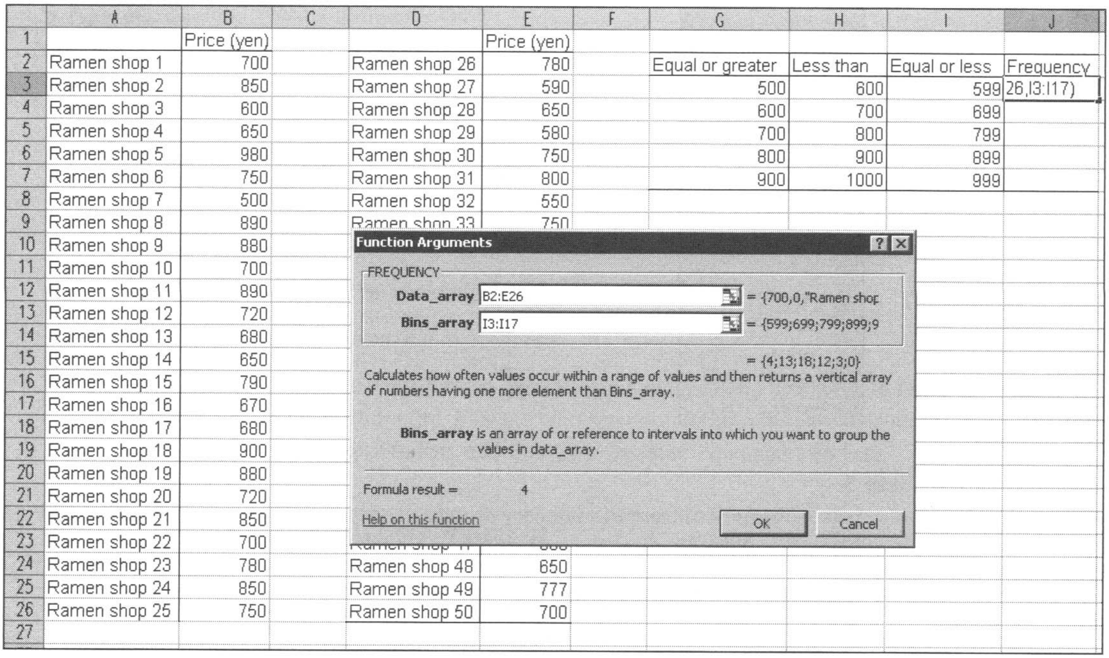
FISHER
FISHERINV
FORECAST
FREQUENCY
FTEST
GAMMADIST
GAMMAINV

FREQUENCY(data_array,bins_array)

Calculates how often values occur within a range of values and then returns a vertical array of numbers having one more element than Bins_array.

Step 4

Select the area shown in the figure below, and click **OK**.



	A	B	C	D	E	F	G	H	I	J
1		Price (yen)			Price (yen)					
2	Ramen shop 1	700		Ramen shop 26	780		Equal or greater	Less than	Equal or less	Frequency
3	Ramen shop 2	850		Ramen shop 27	590		500	600	599	26,I3:I17)
4	Ramen shop 3	600		Ramen shop 28	650		600	700	699	
5	Ramen shop 4	650		Ramen shop 29	580		700	800	799	
6	Ramen shop 5	980		Ramen shop 30	750		800	900	899	
7	Ramen shop 6	750		Ramen shop 31	800		900	1000	999	
8	Ramen shop 7	500		Ramen shop 32	550					
9	Ramen shop 8	890		Ramen shop 33	750					
10	Ramen shop 9	880								
11	Ramen shop 10	700								
12	Ramen shop 11	890								
13	Ramen shop 12	720								
14	Ramen shop 13	680								
15	Ramen shop 14	650								
16	Ramen shop 15	790								
17	Ramen shop 16	670								
18	Ramen shop 17	680								
19	Ramen shop 18	900								
20	Ramen shop 19	880								
21	Ramen shop 20	720								
22	Ramen shop 21	850								
23	Ramen shop 22	700								
24	Ramen shop 23	780		Ramen shop 48	650					
25	Ramen shop 24	850		Ramen shop 49	777					
26	Ramen shop 25	750		Ramen shop 50	700					
27										

Function Arguments

FREQUENCY

Data_array: B2:E26 = {700,0,"Ramen shop

Bins_array: I3:I17 = {599;699;799;899;9

= {4;13;18;12;3;0}

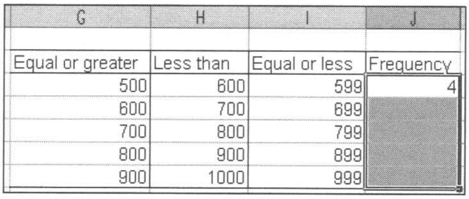
Calculates how often values occur within a range of values and then returns a vertical array of numbers having one more element than Bins_array.

Bins_array is an array of or reference to intervals into which you want to group the values in data_array.

Formula result = 4

Step 5

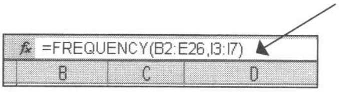
Start with cell J3, and select the area from cell J3 to J7 as shown below.



G	H	I	J
Equal or greater	Less than	Equal or less	Frequency
500	600	599	4
600	700	699	
700	800	799	
800	900	899	
900	1000	999	

Step 6

Click this part in the formula bar.



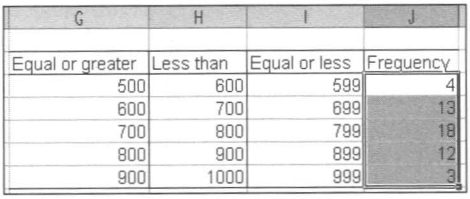
=FREQUENCY(B2:E26,I3:I7)

Step 7

Press ENTER while holding down the SHIFT key and CTRL key at the same time.

Step 8

Now you have the frequency of each class!



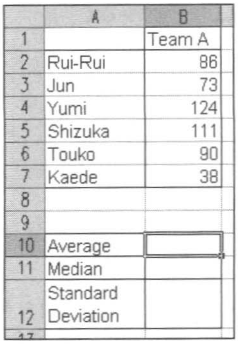
G	H	I	J
Equal or greater	Less than	Equal or less	Frequency
500	600	599	4
600	700	699	13
700	800	799	18
800	900	899	12
900	1000	999	3

2. CALCULATING ARITHMETIC MEAN, MEDIAN, AND STANDARD DEVIATION

This data comes from Rui's classmates' bowling scores on page 41.

Step 1

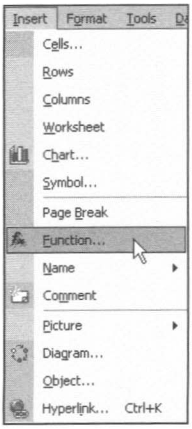
Select cell B10.



	A	B
1		Team A
2	Rui-Rui	86
3	Jun	73
4	Yumi	124
5	Shizuka	111
6	Touko	90
7	Kaede	38
8		
9		
10	Average	
11	Median	
12	Standard Deviation	

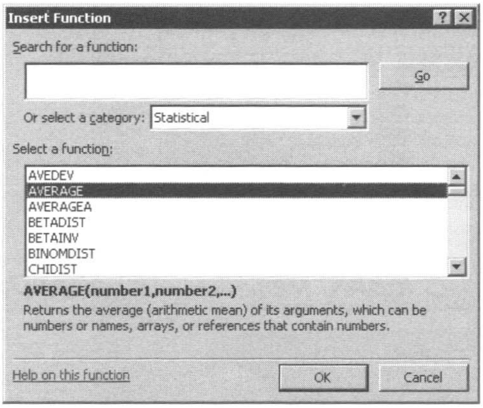
Step 2

Select **Insert ▸ Function**.



Step 3

Select **Statistical** in the category dropdown, and then select **AVERAGE**.



Search for a function:

Or select a category: Statistical

Select a function:

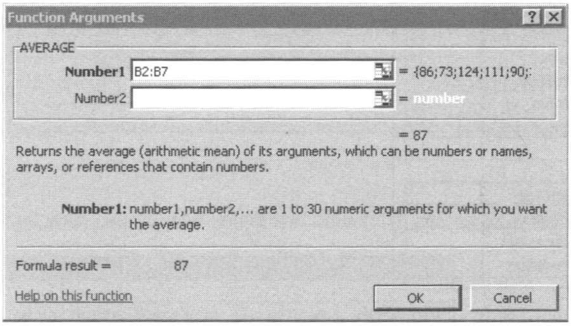
- AVEDEV
- AVERAGE
- AVERAGEA
- BETADIST
- BETAINV
- BINOMDIST
- CHIDIST

AVERAGE(number1,number2,...)

Returns the average (arithmetic mean) of its arguments, which can be numbers or names, arrays, or references that contain numbers.

Step 4

Type the range shown in the figure below, and click **OK**.



AVERAGE

Number1 B2:B7 = {86;73;124;111;90;

Number2 = number

= 87

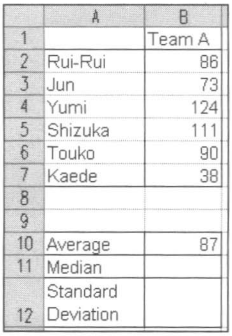
Returns the average (arithmetic mean) of its arguments, which can be numbers or names, arrays, or references that contain numbers.

Number1: number1,number2,... are 1 to 30 numeric arguments for which you want the average.

Formula result = 87

Step 5

Now you have the average score for the team.



	A	B
1		Team A
2	Rui-Rui	86
3	Jun	73
4	Yumi	124
5	Shizuka	111
6	Touko	90
7	Kaede	38
8		
9		
10	Average	87
11	Median	
12	Standard Deviation	

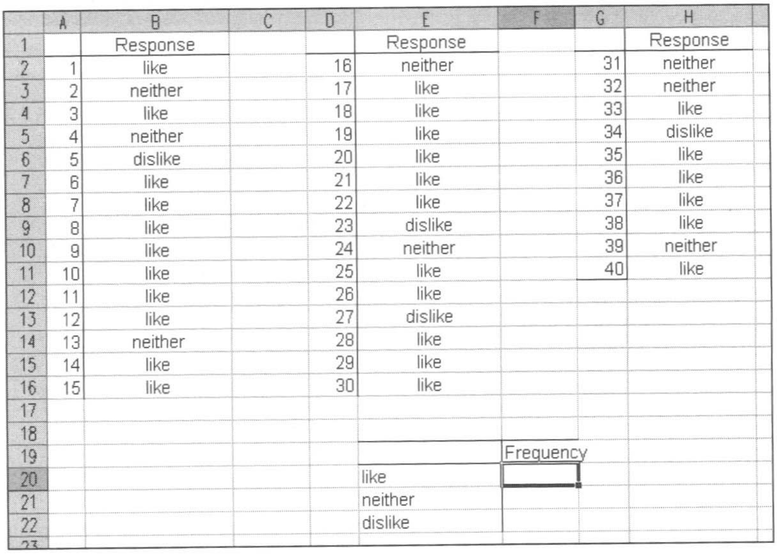
You can calculate the median and standard deviation by following steps 1 through 5 and using the functions MEDIAN and STDEVP in step 2.

3. MAKING A CROSS TABULATION

The data for this table is Rui's classmates' responses to the new uniform design, found on page 61.

Step 1

Select cell F20, then select **Insert ▸ Function**.



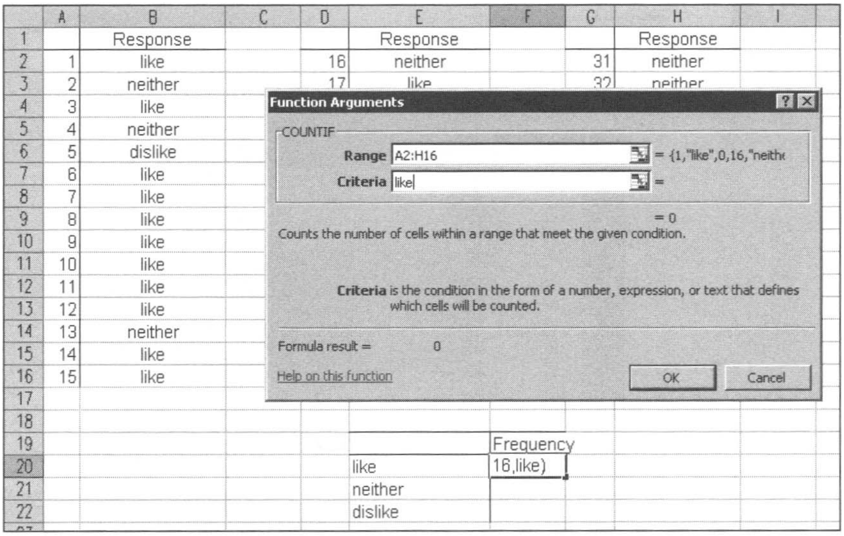
	A	B	C	D	E	F	G	H
1		Response			Response			Response
2	1	like		16	neither		31	neither
3	2	neither		17	like		32	neither
4	3	like		18	like		33	like
5	4	neither		19	like		34	dislike
6	5	dislike		20	like		35	like
7	6	like		21	like		36	like
8	7	like		22	like		37	like
9	8	like		23	dislike		38	like
10	9	like		24	neither		39	neither
11	10	like		25	like		40	like
12	11	like		26	like			
13	12	like		27	dislike			
14	13	neither		28	like			
15	14	like		29	like			
16	15	like		30	like			
17								
18								
19						Frequency		
20					like			
21					neither			
22					dislike			

Step 2

Select **Statistical** in the category dropdown, and then select **COUNTIF** as the name of the function.

Step 3

Select the area shown in the figure below, type *like* in the Criteria text box, and then click **OK**.



	A	B	C	D	E	F	G	H	I
1		Response			Response			Response	
2	1	like		16	neither		31	neither	
3	2	neither		17	like		32	neither	
4	3	like							
5	4	neither							
6	5	dislike							
7	6	like							
8	7	like							
9	8	like							
10	9	like							
11	10	like							
12	11	like							
13	12	like							
14	13	neither							
15	14	like							
16	15	like							
17									
18									
19						Frequency			
20					like	16,like)			
21					neither				
22					dislike				

Function Arguments

COUNTIF

Range A2:H16 = {1,"like",0,16,"neithe

Criteria like =

= 0

Counts the number of cells within a range that meet the given condition.

Criteria is the condition in the form of a number, expression, or text that defines which cells will be counted.

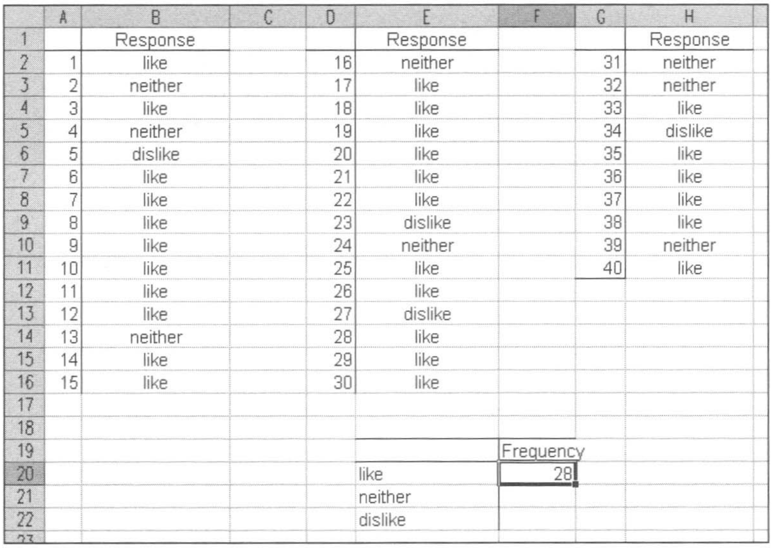
Formula result = 0

Help on this function

OK Cancel

Step 4

Now you have the total number of Rui's classmates who like the new uniform.



	A	B	C	D	E	F	G	H
1		Response			Response			Response
2	1	like		16	neither		31	neither
3	2	neither		17	like		32	neither
4	3	like		18	like		33	like
5	4	neither		19	like		34	dislike
6	5	dislike		20	like		35	like
7	6	like		21	like		36	like
8	7	like		22	like		37	like
9	8	like		23	dislike		38	like
10	9	like		24	neither		39	neither
11	10	like		25	like		40	like
12	11	like		26	like			
13	12	like		27	dislike			
14	13	neither		28	like			
15	14	like		29	like			
16	15	like		30	like			
17								
18								
19						Frequency		
20					like	28		
21					neither			
22					dislike			

Step 5

You can obtain the frequency of *neither* and *dislike* by following steps 1 through 4 and typing those words instead of *like* in step 3.

4. CALCULATING THE STANDARD SCORE AND THE DEVIATION SCORE

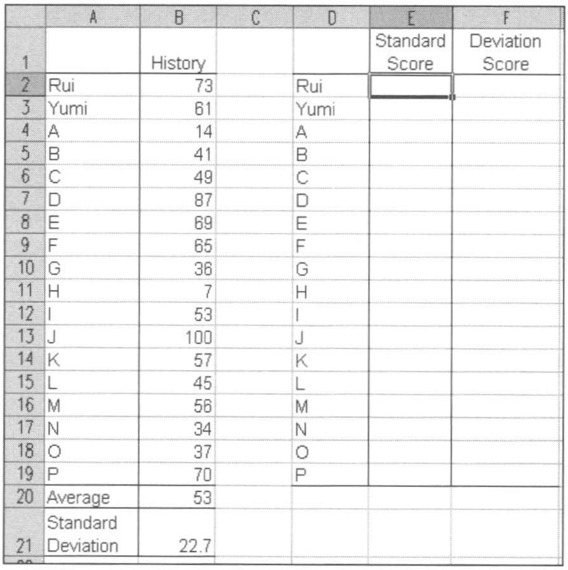
This exercise uses the test data from page 72.

Steps 1 through 8 show the process for obtaining the standard score.

Steps 9 through 11 show the process for obtaining the deviation score. There is an Excel function for calculating standard score, but there is no function for calculating deviation score. However, the deviation score can be calculated fairly easily if we use the result of the standard score calculation.

Step 1

Select cell E2.



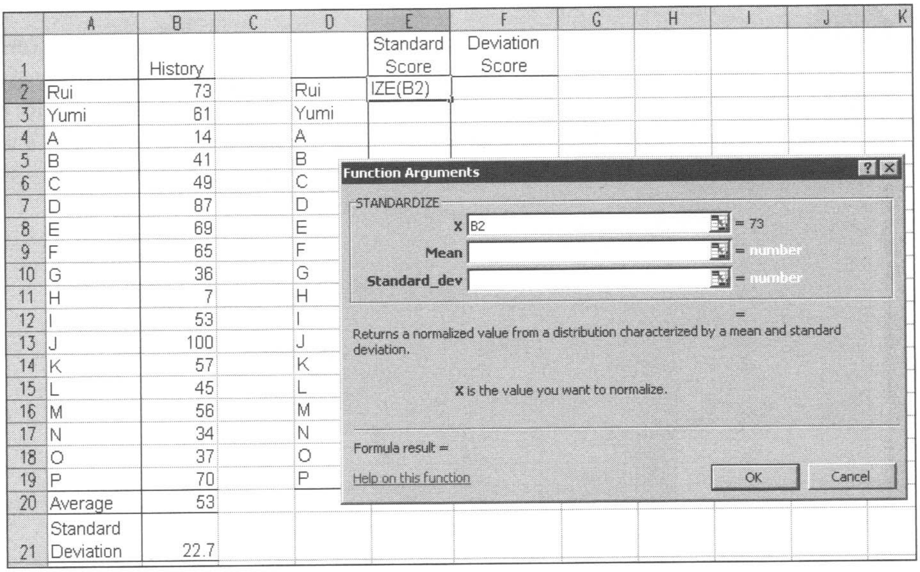
	A	B	C	D	E	F
1		History			Standard Score	Deviation Score
2	Rui	73		Rui		
3	Yumi	61		Yumi		
4	A	14		A		
5	B	41		B		
6	C	49		C		
7	D	87		D		
8	E	69		E		
9	F	65		F		
10	G	36		G		
11	H	7		H		
12	I	53		I		
13	J	100		J		
14	K	57		K		
15	L	45		L		
16	M	56		M		
17	N	34		N		
18	O	37		O		
19	P	70		P		
20	Average	53				
21	Standard Deviation	22.7				

Step 2

Select **Insert ▸ Function**. Then select **Statistical**, and then select **STANDARDIZE** as the name of the function.

Step 3

Select cell B2.



	A	B	C	D	E	F	G	H	I	J	K
1		History			Standard Score	Deviation Score					
2	Rui	73		Rui	IZE(B2)						
3	Yumi	61		Yumi							
4	A	14		A							
5	B	41		B							
6	C	49		C							
7	D	87		D							
8	E	69		E							
9	F	65		F							
10	G	36		G							
11	H	7		H							
12	I	53		I							
13	J	100		J							
14	K	57		K							
15	L	45		L							
16	M	56		M							
17	N	34		N							
18	O	37		O							
19	P	70		P							
20	Average	53									
21	Standard Deviation	22.7									

Function Arguments

STANDARDIZE

X B2 = 73

Mean = number

Standard_dev = number

=

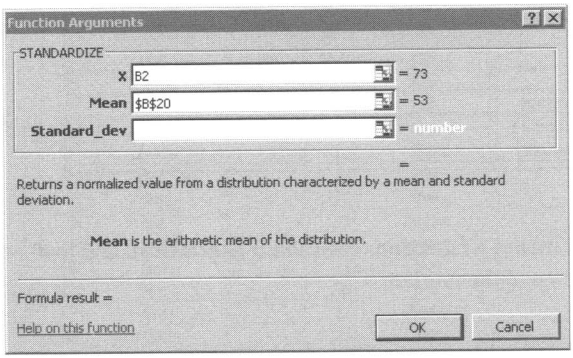
Returns a normalized value from a distribution characterized by a mean and standard deviation.

X is the value you want to normalize.

Formula result =

Step 4

Select B20 for Mean, press F4 once, and confirm that B20 has changed to B20.



Function Arguments

STANDARDIZE

X B2 = 73

Mean B20 = 53

Standard_dev = number

=

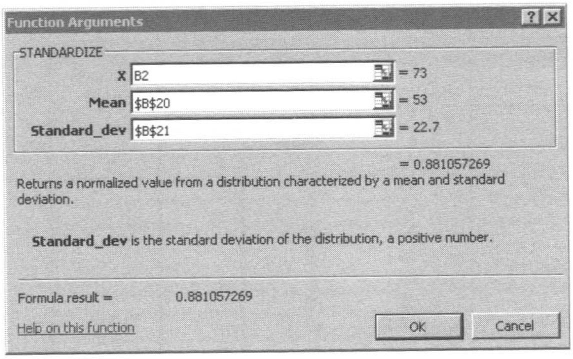
Returns a normalized value from a distribution characterized by a mean and standard deviation.

Mean is the arithmetic mean of the distribution.

Formula result =

Step 5

Select cell B21 for Standard_dev, press F4 once, and after confirming that B21 has changed to B21, click **OK**.



X B2 = 73

Mean B20 = 53

Standard_dev B21 = 22.7

= 0.881057269

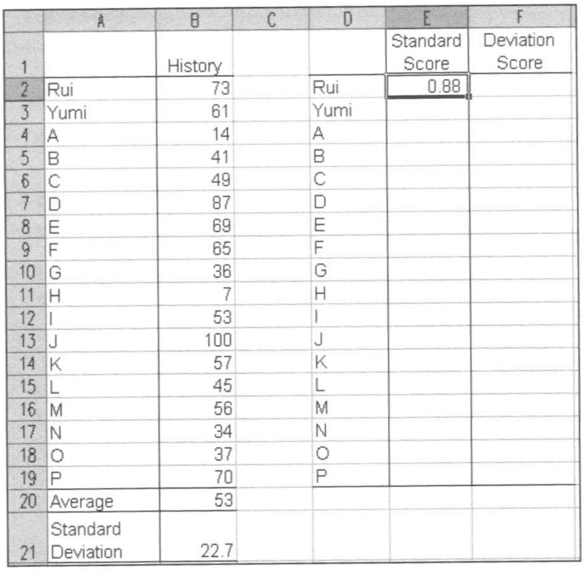
Returns a normalized value from a distribution characterized by a mean and standard deviation.

Standard_dev is the standard deviation of the distribution, a positive number.

Formula result = 0.881057269

Step 6

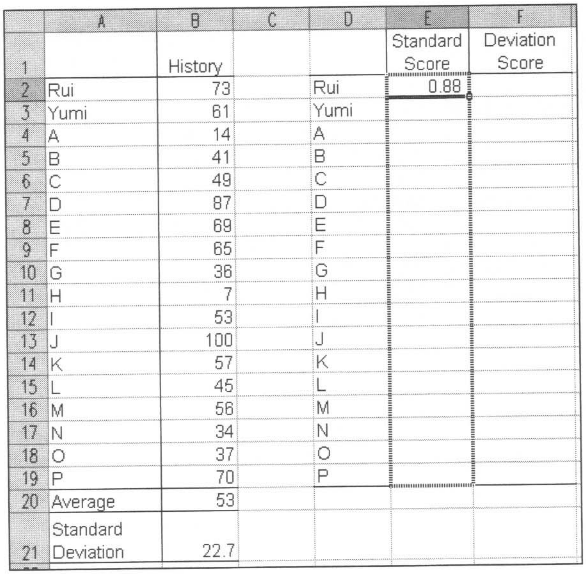
Confirm that Rui's standard score has been calculated.



	A	B	C	D	E	F
1		History			Standard Score	Deviation Score
2	Rui	73		Rui	0.88	
3	Yumi	61		Yumi		
4	A	14		A		
5	B	41		B		
6	C	49		C		
7	D	87		D		
8	E	69		E		
9	F	65		F		
10	G	36		G		
11	H	7		H		
12	I	53		I		
13	J	100		J		
14	K	57		K		
15	L	45		L		
16	M	56		M		
17	N	34		N		
18	O	37		O		
19	P	70		P		
20	Average	53				
21	Standard Deviation	22.7				

Step 7

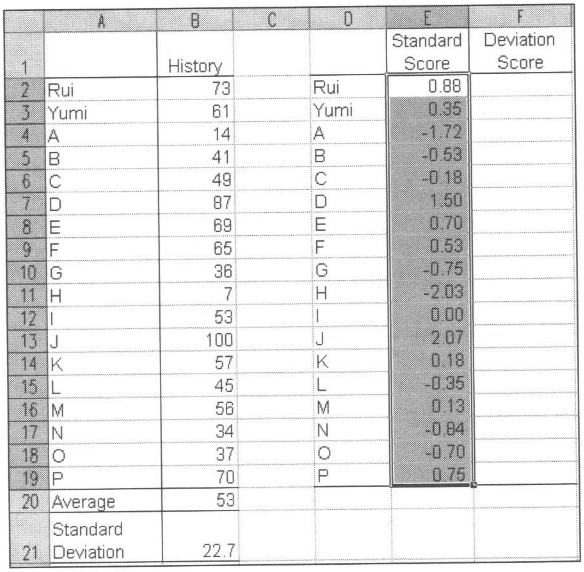
Put the point of the arrow near the bottom-right side of cell E2, confirm that the arrow has changed to a black cross, drag down to cell E19 by holding down the left button of the mouse, and let go of the button when you finish dragging.



	A	B	C	D	E	F
1		History			Standard Score	Deviation Score
2	Rui	73		Rui	0.88	
3	Yumi	61		Yumi		
4	A	14		A		
5	B	41		B		
6	C	49		C		
7	D	87		D		
8	E	69		E		
9	F	65		F		
10	G	36		G		
11	H	7		H		
12	I	53		I		
13	J	100		J		
14	K	57		K		
15	L	45		L		
16	M	56		M		
17	N	34		N		
18	O	37		O		
19	P	70		P		
20	Average	53				
21	Standard Deviation	22.7				

Step 8

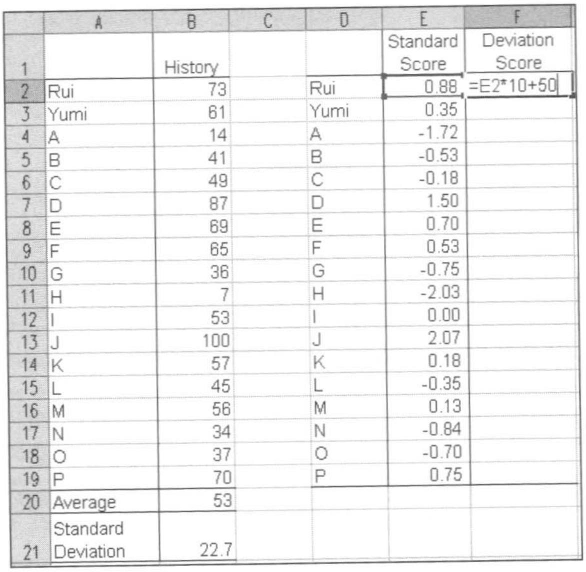
Now you should have everyone's standard score!



	A	B	C	D	E	F
1		History			Standard Score	Deviation Score
2	Rui	73		Rui	0.88	
3	Yumi	61		Yumi	0.35	
4	A	14		A	-1.72	
5	B	41		B	-0.53	
6	C	49		C	-0.18	
7	D	87		D	1.50	
8	E	69		E	0.70	
9	F	65		F	0.53	
10	G	36		G	-0.75	
11	H	7		H	-2.03	
12	I	53		I	0.00	
13	J	100		J	2.07	
14	K	57		K	0.18	
15	L	45		L	-0.35	
16	M	56		M	0.13	
17	N	34		N	-0.84	
18	O	37		O	-0.70	
19	P	70		P	0.75	
20	Average	53				
21	Standard Deviation	22.7				

Step 9

Select cell F2 and type *=E2*10+50*, then press ENTER.



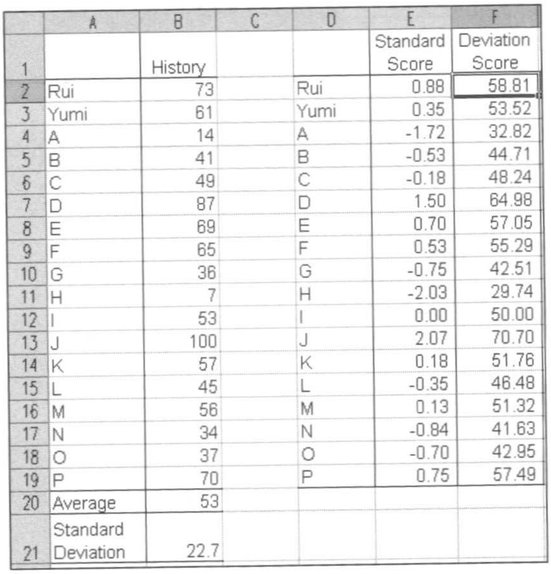
	A	B	C	D	E	F
1		History			Standard Score	Deviation Score
2	Rui	73		Rui	0.88	=E2*10+50
3	Yumi	61		Yumi	0.35	
4	A	14		A	-1.72	
5	B	41		B	-0.53	
6	C	49		C	-0.18	
7	D	87		D	1.50	
8	E	69		E	0.70	
9	F	65		F	0.53	
10	G	36		G	-0.75	
11	H	7		H	-2.03	
12	I	53		I	0.00	
13	J	100		J	2.07	
14	K	57		K	0.18	
15	L	45		L	-0.35	
16	M	56		M	0.13	
17	N	34		N	-0.84	
18	O	37		O	-0.70	
19	P	70		P	0.75	
20	Average	53				
21	Standard Deviation	22.7				

Step 10

Drag down to cell F19, as you did in step 7.

Step 11

Now you have the class's deviation score.



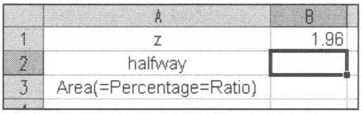
	A	B	C	D	E	F
1		History			Standard Score	Deviation Score
2	Rui	73		Rui	0.88	58.81
3	Yumi	61		Yumi	0.35	53.52
4	A	14		A	-1.72	32.82
5	B	41		B	-0.53	44.71
6	C	49		C	-0.18	48.24
7	D	87		D	1.50	64.98
8	E	69		E	0.70	57.05
9	F	65		F	0.53	55.29
10	G	36		G	-0.75	42.51
11	H	7		H	-2.03	29.74
12	I	53		I	0.00	50.00
13	J	100		J	2.07	70.70
14	K	57		K	0.18	51.76
15	L	45		L	-0.35	46.48
16	M	56		M	0.13	51.32
17	N	34		N	-0.84	41.63
18	O	37		O	-0.70	42.95
19	P	70		P	0.75	57.49
20	Average	53				
21	Standard Deviation	22.7				

5. CALCULATING THE PROBABILITY OF THE STANDARD NORMAL DISTRIBUTION

For this example, we'll use the data from page 93.

Step 1

Select cell B2.



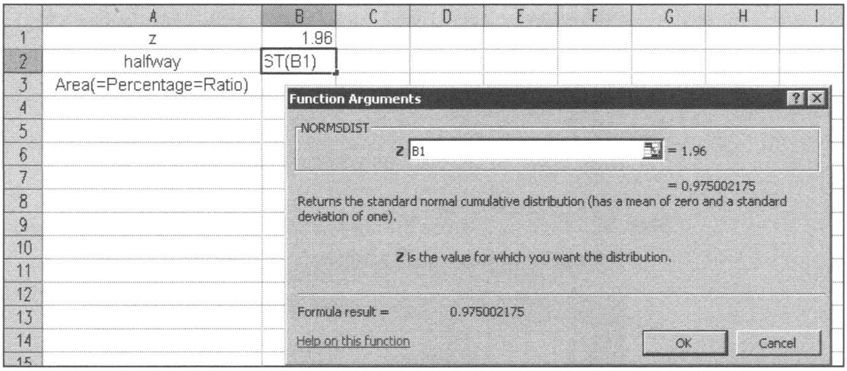
	A	B
1	z	1.96
2	halfway	
3	Area(=Percentage=Ratio)	

Step 2

Select **Insert ▸ Function**, then select **Statistical**, and then select **NORMSDIST**.

Step 3

Select cell B1, and click **OK**.



	A	B
1	z	1.96
2	halfway	ST(B1)
3	Area(=Percentage=Ratio)	

Function Arguments

NORMSDIST

Z B1 = 1.96

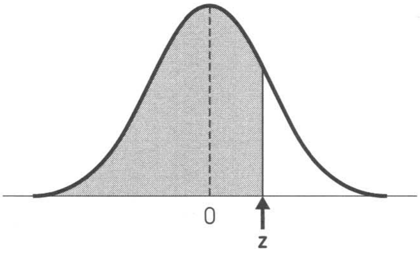
= 0.975002175

Returns the standard normal cumulative distribution (has a mean of zero and a standard deviation of one).

Z is the value for which you want the distribution.

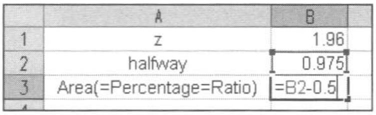
Formula result = 0.975002175

In fact, NORMSDIST is a function to calculate the probability shown in the figure below.



Step 4

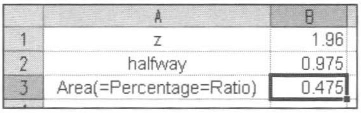
Type *=B2-0.5* in cell B3.



	A	B
1	z	1.96
2	halfway	0.975
3	Area(=Percentage=Ratio)	=B2-0.5

Step 5

Now you have the area.



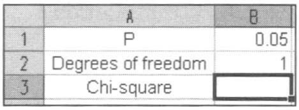
	A	B
1	z	1.96
2	halfway	0.975
3	Area(=Percentage=Ratio)	0.475

6. CALCULATING THE POINT ON THE HORIZONTAL AXIS OF THE CHI-SQUARE DISTRIBUTION

The data for this exercise comes from page 104.

Step 1

Select cell B3.



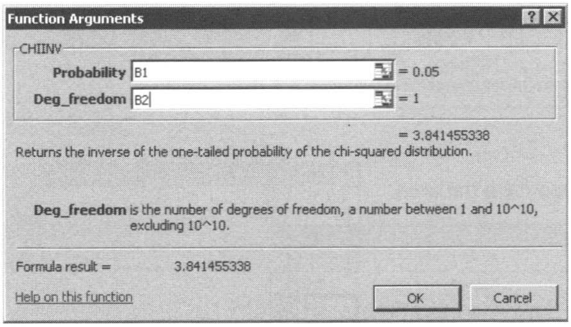
	A	B
1	P	0.05
2	Degrees of freedom	1
3	Chi-square	

Step 2

Select **Insert ▸ Function**, then select **Statistical**, and then select **CHIINV**.

Step 3

Select cells B1 and B2, and then click **OK**.



Function Arguments

CHIINV

Probability: B1 = 0.05

Deg_freedom: B2 = 1

= 3.841455338

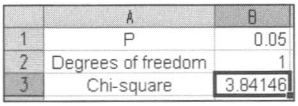
Returns the inverse of the one-tailed probability of the chi-squared distribution.

Deg_freedom is the number of degrees of freedom, a number between 1 and 10^10, excluding 10^10.

Formula result = 3.841455338

Step 4

Now you're done.



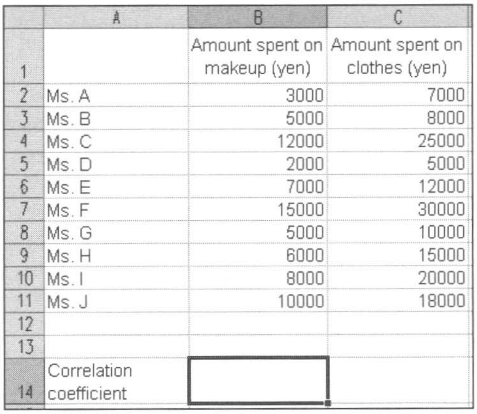
	A	B
1	P	0.05
2	Degrees of freedom	1
3	Chi-square	3.84146

7. CALCULATING THE CORRELATION COEFFICIENT

This data comes from the *P-Girls Magazine* survey found on page 116.

Step 1

Select cell B14.



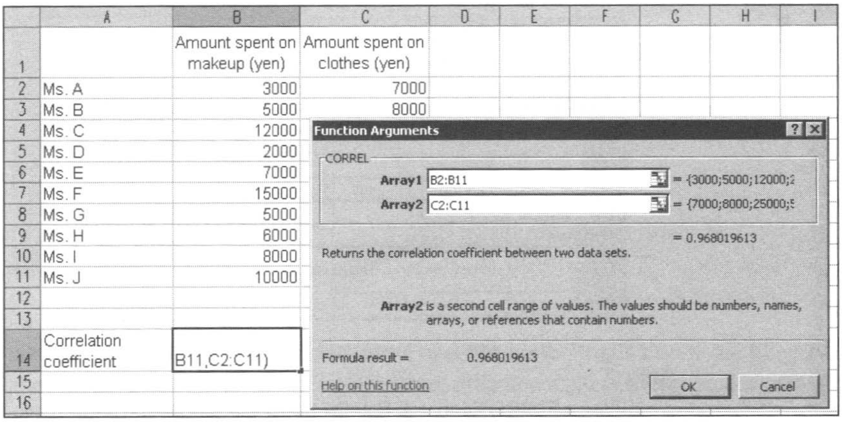
	A	B	C
1		Amount spent on makeup (yen)	Amount spent on clothes (yen)
2	Ms. A	3000	7000
3	Ms. B	5000	8000
4	Ms. C	12000	25000
5	Ms. D	2000	5000
6	Ms. E	7000	12000
7	Ms. F	15000	30000
8	Ms. G	5000	10000
9	Ms. H	6000	15000
10	Ms. I	8000	20000
11	Ms. J	10000	18000
12			
13			
14	Correlation coefficient		

Step 2

Select **Insert ▸ Function**, then select **Statistical**, and then select **CORREL**.

Step 3

Select the area shown in the figure below, and then click **OK**.



	A	B	C
1		Amount spent on makeup (yen)	Amount spent on clothes (yen)
2	Ms. A	3000	7000
3	Ms. B	5000	8000
4	Ms. C	12000	
5	Ms. D	2000	
6	Ms. E	7000	
7	Ms. F	15000	
8	Ms. G	5000	
9	Ms. H	6000	
10	Ms. I	8000	
11	Ms. J	10000	
12			
13			
14	Correlation coefficient	B11,C2:C11)	
15			
16			

Function Arguments

CORREL

Array1 B2:B11 = {3000;5000;12000;2

Array2 C2:C11 = {7000;8000;25000;5

= 0.968019613

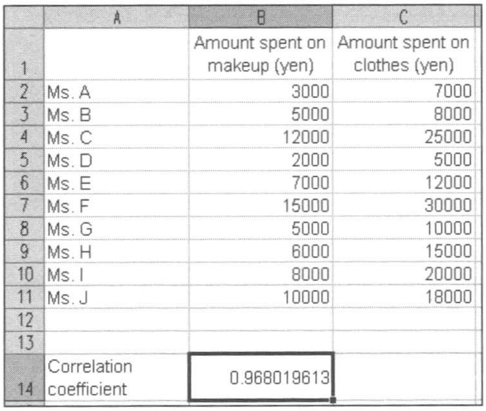
Returns the correlation coefficient between two data sets.

Array2 is a second cell range of values. The values should be numbers, names, arrays, or references that contain numbers.

Formula result = 0.968019613

Step 4

Now you have the correlation coefficient.



	A	B	C
1		Amount spent on makeup (yen)	Amount spent on clothes (yen)
2	Ms. A	3000	7000
3	Ms. B	5000	8000
4	Ms. C	12000	25000
5	Ms. D	2000	5000
6	Ms. E	7000	12000
7	Ms. F	15000	30000
8	Ms. G	5000	10000
9	Ms. H	6000	15000
10	Ms. I	8000	20000
11	Ms. J	10000	18000
12			
13			
14	Correlation coefficient	0.968019613	

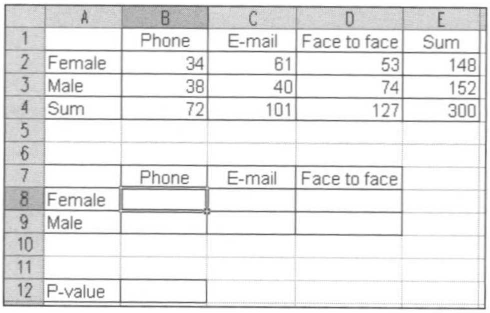
NOTE *Unfortunately, there are no Excel functions for calculating the correlation ratio or the Cramer's coefficient.*

8. PERFORMING TESTS OF INDEPENDENCE

This data is from the dating survey on page 157.

Step 1

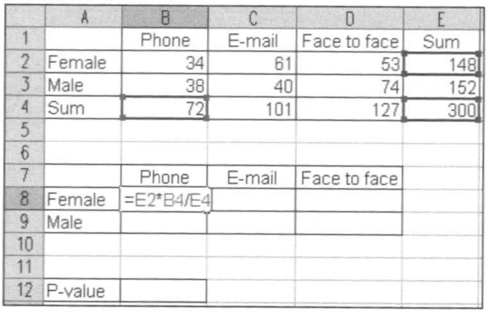
Select cell B8.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	127	300
5					
6					
7		Phone	E-mail	Face to face	
8	Female				
9	Male				
10					
11					
12	P-value				

Step 2

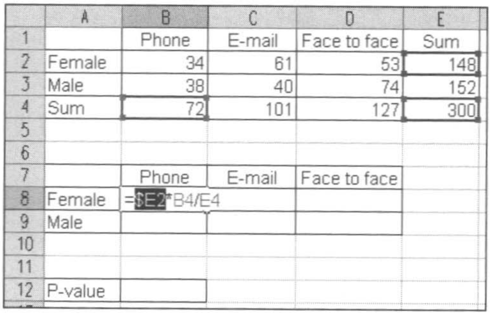
Type *=E2*B4/E4* in cell B8. Do not press ENTER yet.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	127	300
5					
6					
7		Phone	E-mail	Face to face	
8	Female	=E2*B4/E4			
9	Male				
10					
11					
12	P-value				

Step 3

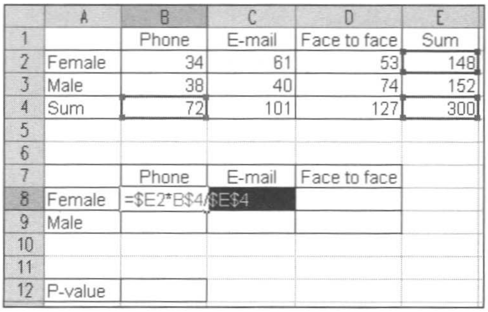
Select *E2* in the equation you just typed, press F4 three times, and confirm that *E2* has changed to *$E2*. Do not press ENTER yet.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	127	300
5					
6					
7		Phone	E-mail	Face to face	
8	Female	=$E2*B4/E4			
9	Male				
10					
11					
12	P-value				

Step 4

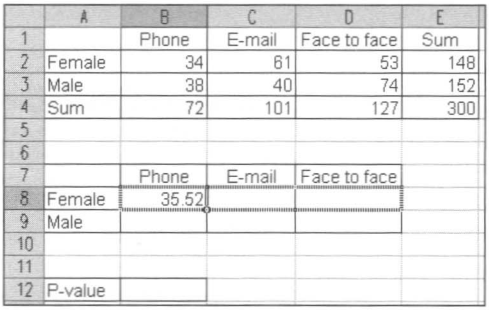
Select *B4* in the equation in cell B8, press F4 twice, and confirm that *B4* has changed to *B$4*. Select *E4* in the equation in cell B8, press F4 once, and confirm that *E4* has changed to *E4*. Then press ENTER.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	127	300
5					
6					
7		Phone	E-mail	Face to face	
8	Female	=$E2*B$4/E4			
9	Male				
10					
11					
12	P-value				

Step 5

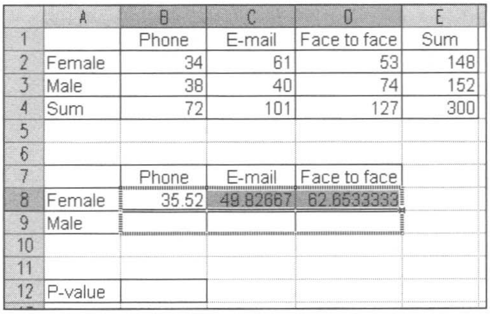
Select cell B8, put the point of the arrow near the bottom right side of cell B8, confirm that the arrow has changed to a black cross, drag down to cell D8 by holding down the left button of the mouse, and let go of the button when you finish dragging.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	127	300
5					
6					
7		Phone	E-mail	Face to face	
8	Female	35.52			
9	Male				
10					
11					
12	P-value				

Step 6

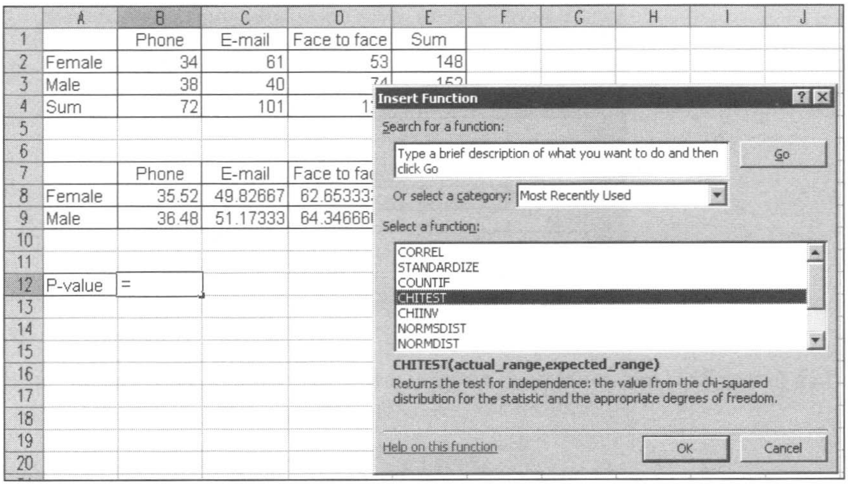
Select the area from cell B8 to D8, put the point of the arrow near the bottom right side of cell D8, confirm that the arrow has changed to a black cross, drag down to cell D9 by holding down the left button of the mouse, and let go of the button when you finish dragging.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	127	300
5					
6					
7		Phone	E-mail	Face to face	
8	Female	35.52	49.82667	62.6533333	
9	Male				
10					
11					
12	P-value				

Step 7

Select cell B12, select **Insert ▸ Function**, then select **Statistical**, and then select **CHITEST**.



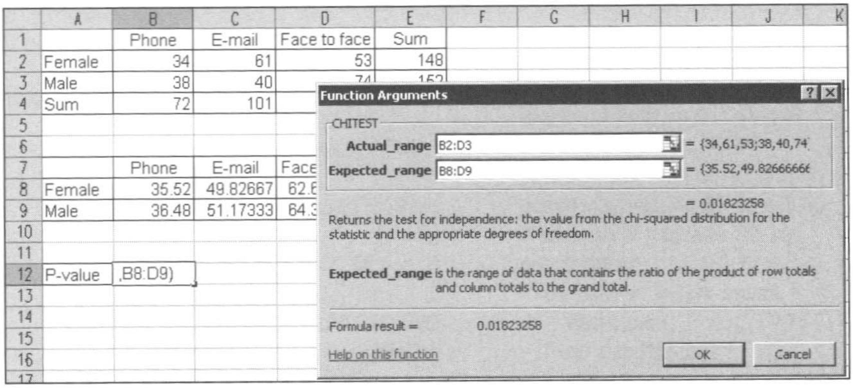
	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	1	
5					
6					
7		Phone	E-mail	Face to fac	
8	Female	35.52	49.82667	62.65333	
9	Male	36.48	51.17333	64.34666	
10					
11					
12	P-value	=			

CHITEST(actual_range,expected_range)

Returns the test for independence: the value from the chi-squared distribution for the statistic and the appropriate degrees of freedom.

Step 8

Select the area shown in the figure below, and then click **OK**.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40		
4	Sum	72	101		
5					
6					
7		Phone	E-mail	Face	
8	Female	35.52	49.82667	62.6	
9	Male	36.48	51.17333	64.3	
10					
11					
12	P-value	,B8:D9)			

Function Arguments

CHITEST

Actual_range B2:D3 = {34,61,53;38,40,74

Expected_range B8:D9 = {35.52,49.8266666

= 0.01823258

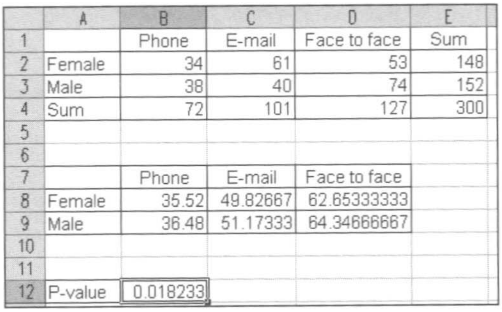
Returns the test for independence: the value from the chi-squared distribution for the statistic and the appropriate degrees of freedom.

Expected_range is the range of data that contains the ratio of the product of row totals and column totals to the grand total.

Formula result = 0.01823258

Step 9

Now you're done. You can confirm that the calculated value is equal to the P-value on page 177.



	A	B	C	D	E
1		Phone	E-mail	Face to face	Sum
2	Female	34	61	53	148
3	Male	38	40	74	152
4	Sum	72	101	127	300
5					
6					
7		Phone	E-mail	Face to face	
8	Female	35.52	49.82667	62.65333333	
9	Male	36.48	51.17333	64.34666667	
10					
11					
12	P-value	0.018233			

INDEX